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AN EXPERIMENTAL STUDY OF TAYLOR-GOERTLER VORTICES IN A CURVED RECTANGULAR CHANNEL

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THESIS

AN EXPERIMENTAL STUDY OF TAYLOR-GOERTLER VORTICES IN A CURVED RECTANGULAR CHANNEL

by

David Lee Flentie

March 1975

Thesis Advisor: Matthew Kelleher

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freedom permitted a rectangular cross section of the flow to be investigated. It was concluded that the onset of Taylor-Goertler vortices was in agreement with existing information that the intensity of the vortices increased with increasing flow rate, and turbulent fluctuations grew within the vortex structure. It was also concluded that flow in a curved channel resulted in distortion of the normally parabolic velocity profile shifting the maximum mean velocity toward the concave wall.

An Experimental Study of Taylor-Goertler Vortices in a Curved Rectangular Channel

by

David Lee Flentie
Lieutenant Commander, United States Navy
B.S., United States Naval Academy, 1964

Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

A rectangular cross section channel, designed and built by R. McKee to create the laminar secondary Taylor-Goertler vortex flow, was modified to expand the experimental investigation of the vortex flow. The modification allowed the hot wire anemometer, used to obtain velocity profiles, to be moved in both the radial and transverse direction. This additional freedom permitted a rectangular cross section of the flow to be investigated. It was concluded that the onset of Taylor-Goertler vortices was in agreement with existing information, that the intensity of the vortices increased with increasing flow rate, and turbulent fluctuations grew within the vortex structure. It was also concluded that flow in a curved channel resulted in distortion of the normally parabolic velocity profile shifting the maximum mean velocity toward the concave wall.

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I. INTRODUCTION

A. DESCRIPTION OF TAYLOR-GOERTLER VORTICES

There has been considerable evidence compiled that indicates a fully developed laminar flow along a concave wall section does not remain purely two-dimensional. It has been demonstrated experimentally that such a flow pattern forms a system of counter-rotating vortex pairs (Figures 1 and 2) whose axes are in the direction of mean motion.

This secondary flow pattern is caused by the variation in centrifugal forces on fluid particles at different locations in the flow. The velocity profile of a viscous fluid in fully developed laminar flow between parallel plates is parabolic. Because of this profile, there exists a region at the center of the channel whose velocity is relatively high compared to the velocity of the fluid in close proximity to the concave and convex wall surfaces. The fluid with higher kinetic energy, in the center of the channel, is subjected to greater centrifugal forces and therefore has a tendency to move toward the concave surface. This motion displaces fluid along the concave surface which then moves to the center of the channel and replaces the fluid that has vacated that area. The fluid now in the center of the channel is subjected to the higher flow velocity and the rotation is continued. This cyclic motion forms the counter-rotating Taylor-Goertler vortices.

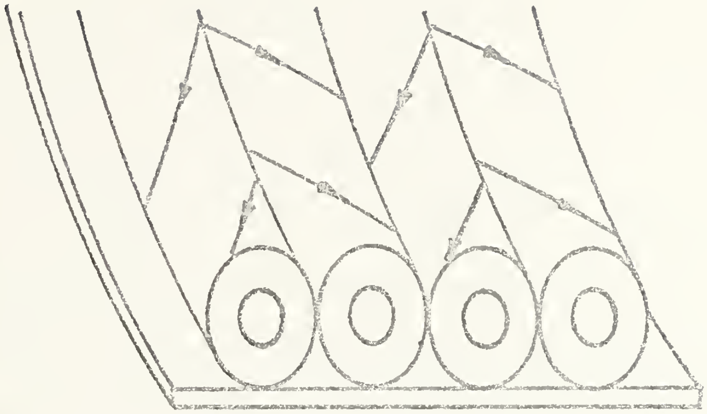

FIGURE 1. Vortex disturbance in the flow of a fluid on a concave wall, axes of the vortices parallel to principal flow direction.

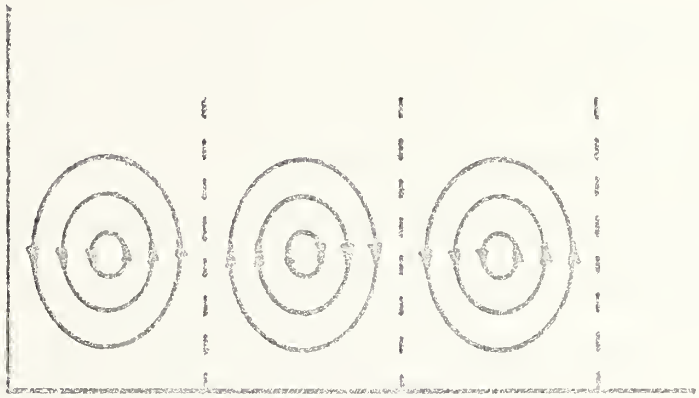

FIGURE 2. Streamline patterns in a section at right angle to the principal flow axis.

B. BRIEF HISTORY

The stability of an invisid fluid moving in concentric layers was first studied by Lord Rayleigh [1] in 1916. In his investigation of flow between rotating cylinders, he determined the stability criterion for invisid flow to be that the product of the local circumferential velocity and the local radius of curvature either increase or at least remain constant as the radius increases. G. I. Taylor [2] continued the work of Rayleigh and examined extensively, both analytically and experimentally, the flow of viscous fluids between rotating cylinders. He found that when the inner cylinder rotates and the outer cylinder remains stationary, the motion becomes unstable when the dimensionless Taylor number,

$$\text{Taylor number} = R_e\sqrt{d/r_c}$$

exceeds a value of about 41. In the equation above, d is the spacing between the cylinders, which is assumed small compared with r_c, the radius of the inner cylinder. R_e is the Reynolds number based on the circumferential velocity of the inner cylinder and the length d. For values of Taylor number greater than 41, the critical value, a secondary motion sets in with the familiar cellular form.

W. Dean [3] was the first to consider the similar type of instability that occurs when a viscous fluid flows in a curved channel due to a pressure gradient acting around the channel. The problem considered by Dean was a channel formed by two concentric cylinders, the spacing between the cylinders

being small when compared to the radius of the inner cylinder. Dean determined that instability will first arise when the Dean number,

$$\text{Dean number} = R_e\sqrt{d/r_c}$$

is greater than 36, where the Reynolds number, R_e, is based on the mean velocity of the unperturbed flow. It was determined that a cellular type pattern of secondary flow vortices would exist similar to those of Taylor's experiments. W. Reid [4] later examined the work of Dean and verified his analytical results by a simplified method.

In 1940, H. Goertler [5] investigated the stability of laminar boundary layer profiles on slightly curved walls relative to small disturbances, in the shape of vortices, whose axes are parallel to the principal direction of flow. Goertler concluded from numerical calculations that amplified disturbances were produced only on the concave walls. Further analysis yielded information about stability limits, ranges of vortex wave lengths that can be amplified, and the transition from laminar to turbulent flow. Goertler's approximations have been confirmed with exact solutions by G. Hammerlin [6] and with extensive numerical analysis by A. M. O. Smith [7].

L. Persen [8], in 1965, investigated the effect of Taylor-Goertler vortices on the transfer of heat across laminar boundary layers for special cases of very high and very low Prandtl numbers. He determined that the overall effect of these vortices was to increase the rate of heat transfer through the boundary layer. It should also be mentioned that

the Scherberg concept, described in private communications between M. Scherberg and L. Persen [19], of local instability fixes attention on the geometry of the stream lines of the flow rather than the geometric configuration of the surroundings in which the flow takes place. (This concept appears to be a restatement of Rayleigh's criterion.) This concept increases the scope of information derived from flows in curved channels enormously. The first experimental work on the effect of Taylor-Goertler vortices was reported by P. McCormack et al. [9] in their 1970 study. Cheng and Akiyama [10], in 1970, calculated the laminar forced convection heat transfer in rectangular channels. They considered channels with various ratios of width to height at values of Dean number up to 500. Analytic and experimental results for a fully developed, constant wall heat flux, square cross sectional area, curved channel flows were obtained by Y. Mori and Y. Uchida [11] in 1967.

Taylor-Goertler vortices are a phenomenon of laminar flow but they affect the transition from laminar to turbulent flow. (See H. Liepmann [12]). These vortices have similarities to other vortex flows such as wing tip vortices and the vortex rolls in forced convection heating of fluid layers described in the paper by M. Akiyama et al. [13]. The striations seen at stagnation points on blunt bodies and the cross-hatching observed on reentry vehicles has been explained, at least in part, by the presence of Taylor-Goertler vortices (See M. Tobak [14]).

There are many possible applications that could result from an improved understanding of Taylor-Goertler vortices. A few such applications could be improved turbine blade cooling, improved surface cooling techniques, and a better understanding of the transition between laminar and turbulent flow. Curved channels occur frequently in fluidics devices and an increased understanding of Taylor-Goertler vortices may lead to design improvements. Heat exchangers could be designed that take advantage of the low pressure drop associated with laminar flow and the improved heat transfer characteristics of the flow with vortices present.

II. NATURE OF THE PROBLEM

A. INTENT OF THIS STUDY

Lieutenant McKee [15], in his Masters Thesis, made a detailed study of the effects of secondary flow on the velocity profile at one point in a curved channel of rectangular cross sectional area. The direction of this study was to continue the work of McKee and follow his recommendations to modify the existing equipment so that the hot wire probe used in his investigation could be moved across the channel in a spanwise direction. In this way, the size and nature of the vortices generated could be investigated. It was also the intent of this study to investigate the flow characteristics at the critical point at which secondary flow develops and the transition from laminar to turbulent flow occurs.

B. DESIGN REQUIREMENTS

The apparatus built by McKee was used for this study with several modifications. The considerations that went into McKee's design were that the limit of stability of the basic laminar flow in a curved channel was that presented by Reid. If the Dean number, defined as

$$D_e = (Ud/\nu)\sqrt{d/r_c} \quad ,$$

exceeds the value of 36, it is presumed that Taylor-Goertler vortices will be present. It was also known that the rate at which the vortices develop is dependent to some degree on how much greater than 36 the Dean number becomes. It has been

determined that the size of the vortices will depend on the dimensions of the channel. Taylor-Goertler vortices on curved channels have varied in size from that of the boundary layer to ten times larger, depending on geometric factors. The upper limit of the Dean number was taken as 150, as it was assumed that this provided a sufficient range for the vortices to become fully developed.

The minimum critical Reynolds number, based on hydraulic diameter, at which turbulent flow is observed in a parallel channel was given by G. Beavers et al. [16] as $R_e = 2200$. It was reported by several investigators, including H. Liepmann and R. Nunge [17], that the decrease in critical Reynolds number due to curvature is small, especially for large radii of curvature. The range of Reynolds number selected for design of the experimental apparatus was from 100 to 2000. It was determined the Reynolds number could be accurately measured throughout this range.

The effects of such variables as the working fluid, the channel height, and the flow rate on the Reynolds number and Dean number were checked. It was realized that the radius of curvature also effected the Dean number. A trial and error method was used to select the height and width of the channel cross section. The following inequality in the Reynolds number was checked to give the range of velocities,

$$100 \leq R_e \leq 2000.$$

The inequality in the Dean number given below was then checked to give the radius of curvature,

$$36 \leq D_e \leq 150.$$

The design variables were considered feasible when the velocity range was practical to obtain and easy to measure, and when the radius of curvature was reasonable for construction. The design selected by McKee had a ratio of radius of curvature to plate spacing of 48. Air was selected as the working fluid over water because water required a much more complex construction to prevent leakage from the channel. Also, the flow velocities at which water satisfied the Reynolds number and the Dean number were too low to measure by ordinary means. The length of the entrance region was checked to insure it was not excessive. An entrance contraction section was designed in accordance with considerations given by M. Cohen and N. Ritchie [18] to insure smooth entering flow. Plexiglas was selected as the material for the walls of the channel because it was workable, transparent, and inexpensive.

It was hoped that the results obtained would be comparable to infinite parallel plate solutions, therefore an aspect ratio of 40 was chosen.

C. DESCRIPTION OF THE APPARATUS

The cross section of the channel was 0.250 inches high and 10.0 inches wide. The aspect ratio was 40 and it had a cross sectional area of 0.017361 square feet. The hydraulic diameter of the channel, defined as

$$D_H = 4A/P,$$

was 0.04065 feet. The channel was made of two one-quarter inch thick sheets of plexiglas separated by rolled metal

spacers which also formed the sides of the channel. The channel was composed of three segments: a four foot long entrance section, the curved section consisting of a 180 degree turn with a radius of curvature of the interior concave wall of one foot, and a short straight section.

The flow of the air was introduced to the channel through a contraction section attached to the beginning of the entrance region and covered by a cheese cloth screen. The flow left the channel through an exhaust nozzle which was connected by flexible tubing to a Fisher and Porter Company variable area flow meter, model number 10A3565. The rotometer had a 100% full scale flow rate of 11.1 standard cubic feet of air per minute and an accuracy of ± 0.5% of full scale. The flow was drawn through the channel by an electrically driven Cadillac, model G12, centrifugal blower. The blower speed was controlled by varying the motor voltage with a variac. The voltage was supplied by a Sorenson, R1050, A.C. voltage regulator.

The measurement of the velocity profile was done by using a sub miniture hot wire anemometer probe, Thermo Systems, Inc. model 1279, which had a 90-degree bend in the supports. This probe was located 29 inches of arc length down stream from the start of curvature and swept along a line from the center line of the channel outward two inches toward the side of the channel (See figure 3). The probe was placed in a plug and inserted through the convex wall of the channel. The inside surface of the plug had the same radius of curvature as that of the channel and it fit flush with the inside surface.

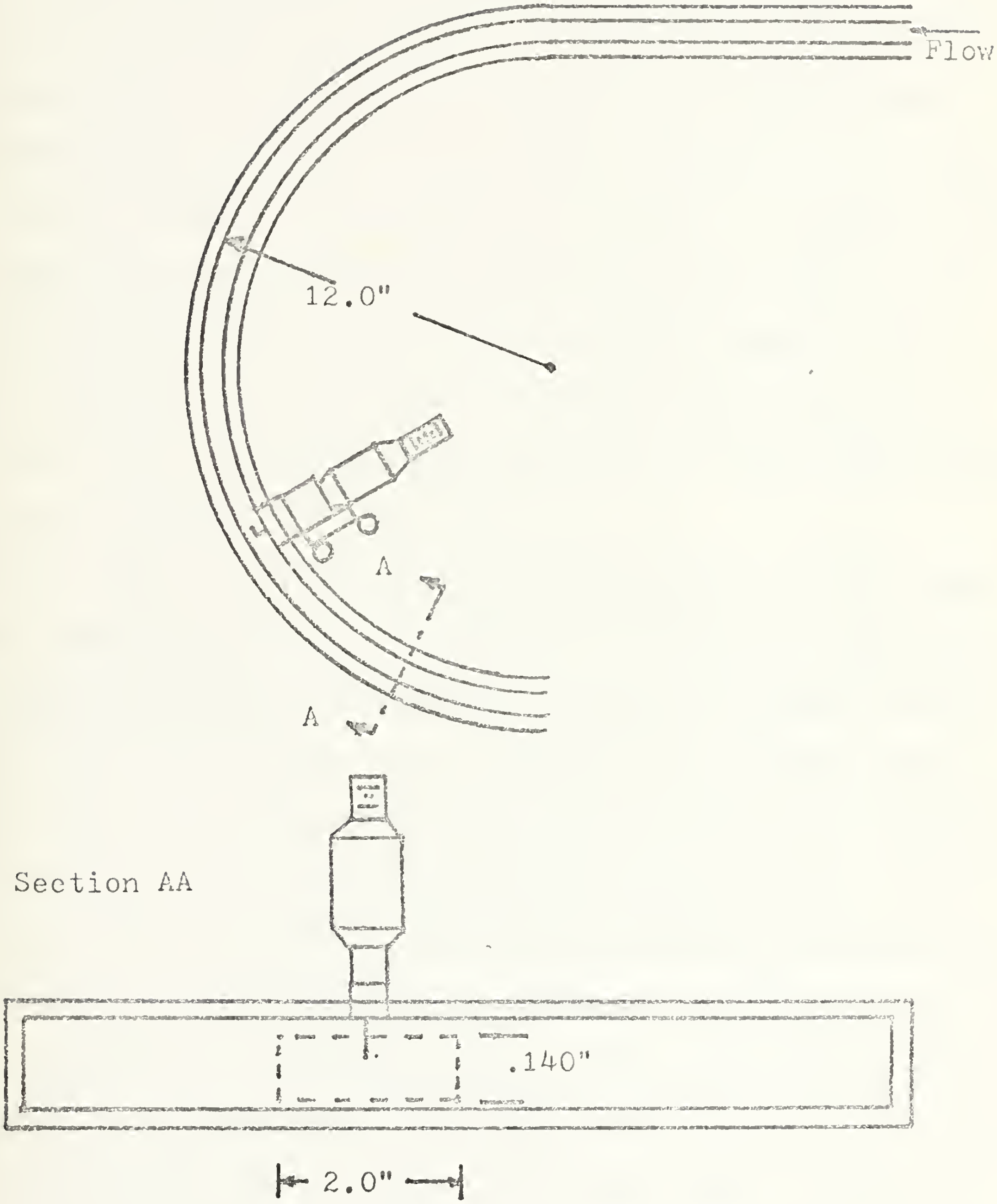

FIGURE 3. Hot wire anemometer arrangement.

The 0.0350 inch diameter body of the hot wire anemometer protruded from the plug and was fastened to a micrometer barrel which was mounted on the traversing assembly. The traversing assembly was driven by a DISA, type 55H01, traversing mechanism and permitted the probe to be driven along a six inch horizontal slot that had been machined in the convex wall of the channel. The plug was attached to a plastic belt, one inch wide and 0.010 inches thick, that ran along a horizontal groove that had been machined on the interior side of the convex wall. The belt was used to seal the exposed portion of the slot from the internal flow as the probe moved from side to side.

The leads from the probe were connected to a DISA, type 55D01, constant temperature anemometer. From the anemometer the signal was processed through a DISA, type 55D10, linearizer, a DISA, type 55D30, D.C. voltmeter and finally input on the Y-axis of a Hewlett-Packard, model 7035B, X-Y recorder. The traversing mechanism provided the X input to the X-Y recorder. The micrometer barrel that permitted the probe to be moved in a radial direction was graduated in 0.001 of an inch. Turbulence was monitored by a DISA, type 55D35, RMS voltmeter and a Tektronix, type 531, oscilloscope. A Thermo-Systems, Inc. calibrator, model 1125, was used to calibrate the hot wire anemometer and to determine the proper constant setting for the linearizer to produce a linear response to the nonlinear output of the probe.

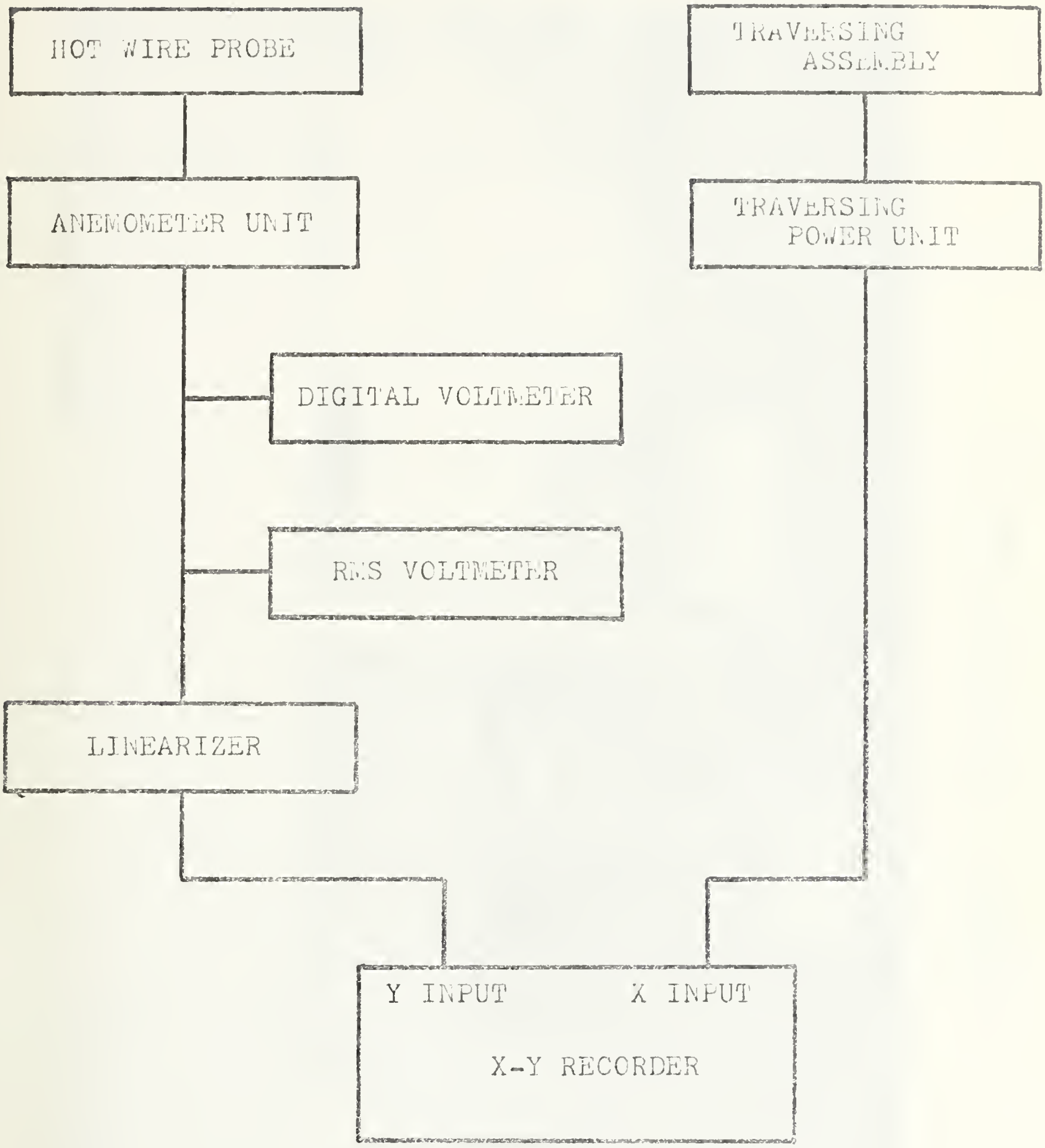

FIGURE 4. Flow diagram of electronics.

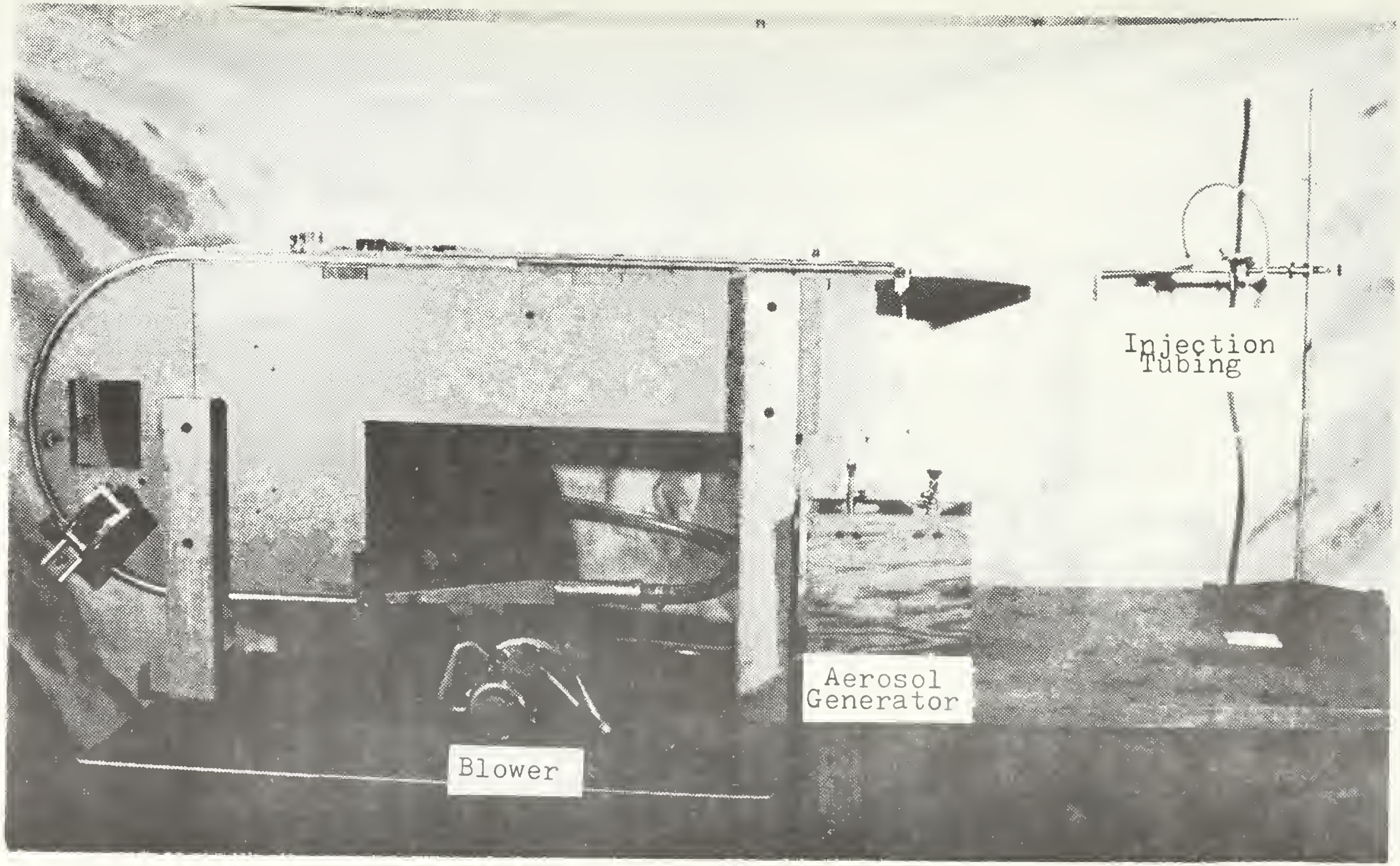

FIGURE 5. Experimental apparatus

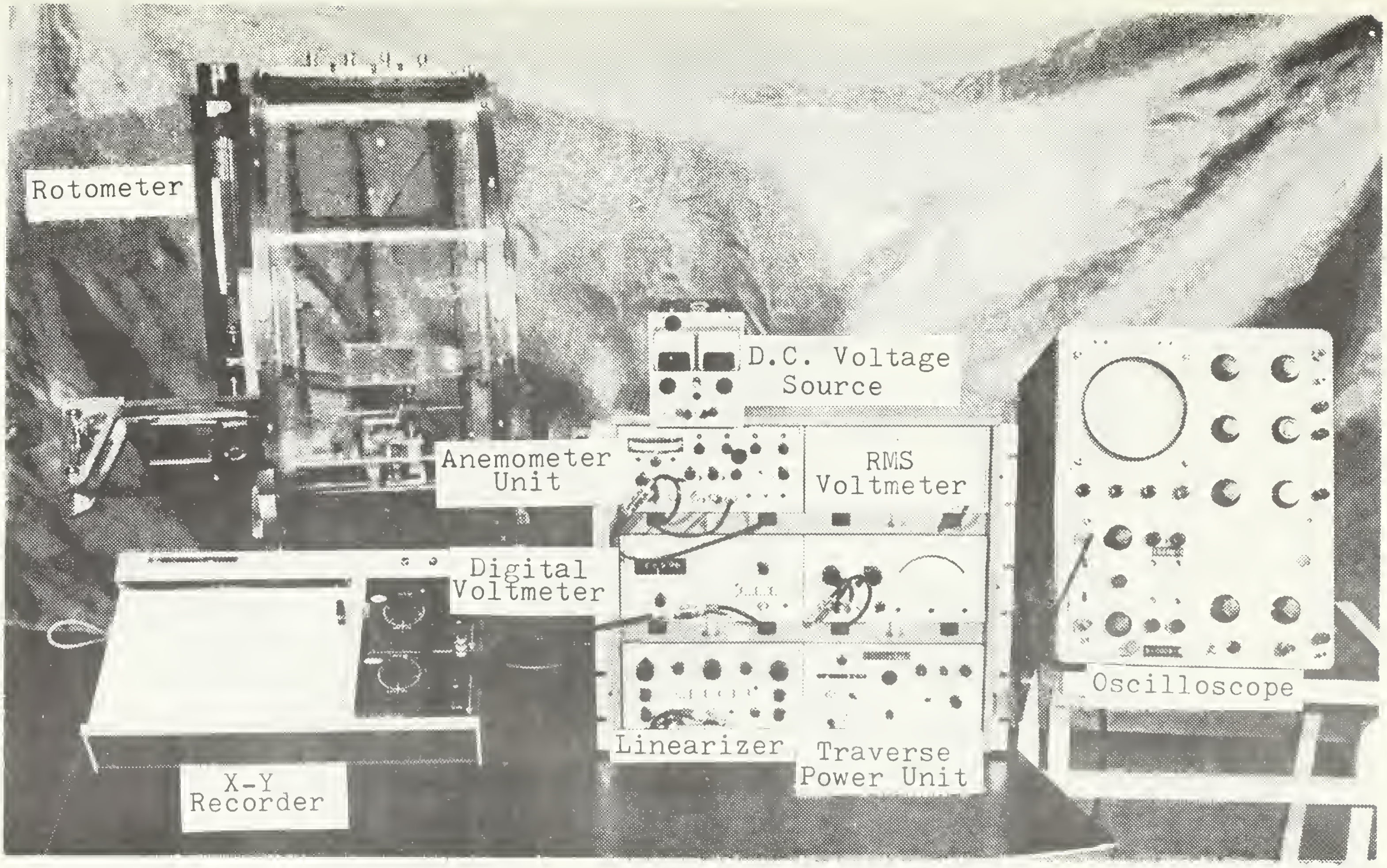

FIGURE 6. Electronic equipment

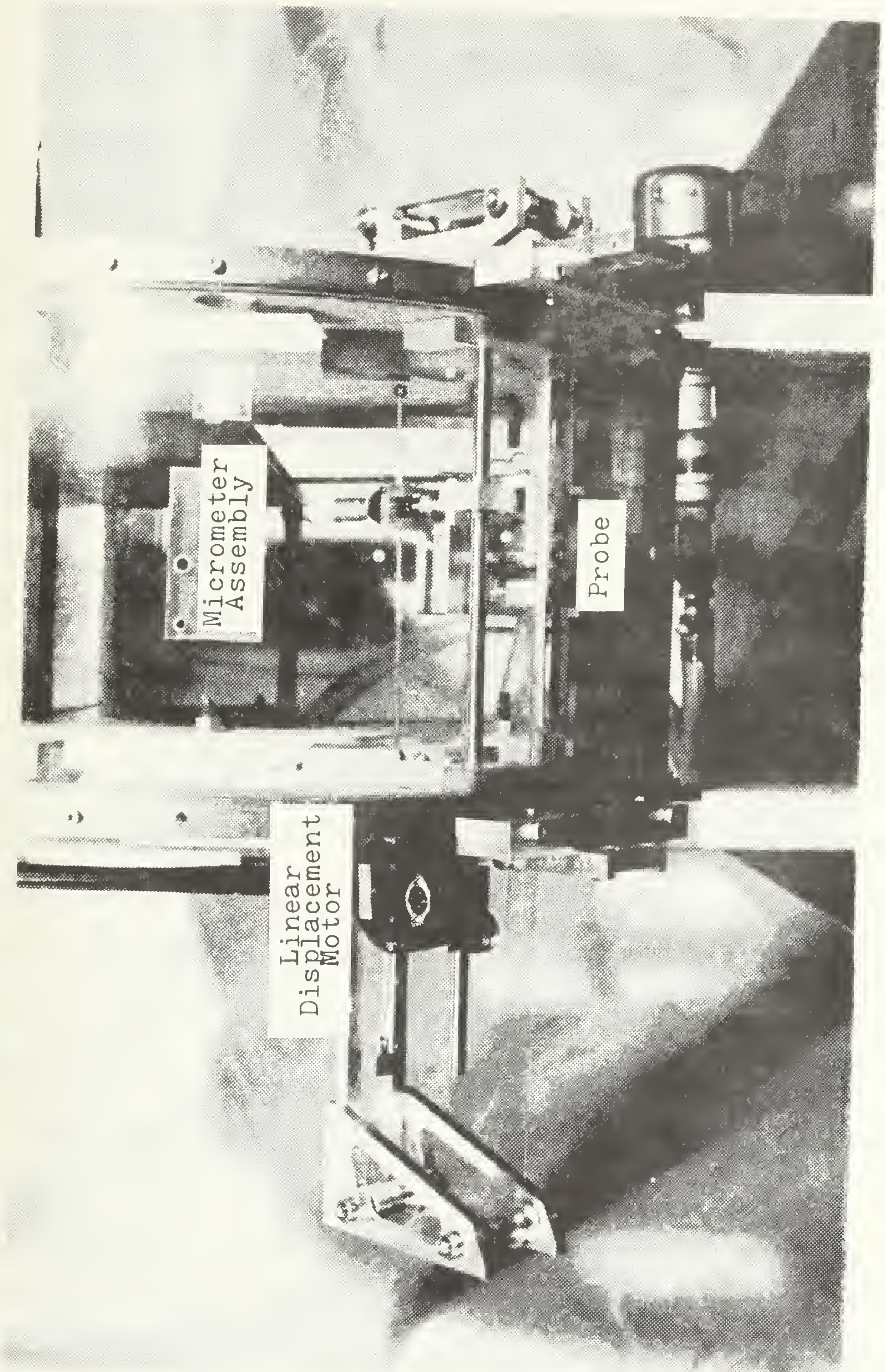

FIGURE 7. Traversing assembly

III. HOT WIRE MEASUREMENTS

A. EXPERIMENTAL PROCEDURES

The sub miniture hot wire anemometer probe that was to be used in the experimental work was connected to the anemometer unit. The cold resistance of the wire was measured as 8.08 ohms. The anemometer's operating resistance was set at a value of 12.12 ohms, based on an overheat ratio of 1.5. The probe was centered in the test chamber of the calibration unit. A filtered air supply was connected to the calibrator and the pressure drop across the calibrator flow nozzle was measured by an inclined manometer. With the anemometer in the operational mode, the flow rate through the calibration unit was adjusted to several different predetermined levels. The output voltage of the bridge in the anemometer unit was connected to a digital voltmeter and to an RMS meter where it was recorded along with the pressure drop in inches of water as indicated by the manometer. This data was reduced by using the tabulated values in the technical manual for the calibration unit. Using the reduced data, the curve plotting mean velocity versus voltage was obtained. The output signal from the anemometer bridge was then connected to the linearizer unit and a trial and error method was then used to determine the correct values to convert the fourth power relationship existing between flow velocity and output voltage of the probe to a linear relationship. It was

determined that the exponent value of 2.10 and gain value of 1.525 set on the linearizer provided the most linear output in the region to be investigated.

The hot wire anemometer probe was removed from the calibration unit and inserted as far into the mounting as possible. The mounting block for the micrometer barrel was attached to the stem of the probe. The range of settings on the micrometer barrel went from 0.500 inches, when the probe was in contact with the convex wall, to 0.250 inches when it was in contact with the concave wall. In order to allow for small irregularities in the wall surface, it was determined that the range of investigation would be over the micrometer variation of 0.400 inches to 0.260 inches. This range would allow the investigation of the prime area of interest, that of the concave wall, to be examined down to 0.01 inches from the surface while protecting the probe from damage caused by slight irregularities on the surfaces.

Prior to taking any data, the blower and all electronic equipment were allowed to warm up for a period of several hours. After sufficient warm up and with the stability of the equipment verified, the vertical and horizontal scales of the X-Y recorder were set using a constant voltage source. Several preliminary runs were made at various flow rates to determine the scales that would produce the best graphic display. A transverse sweep length of two inches was determined to provide an adequate area for investigation.

The first series of data runs was intended to investigate the flow characteristics at four separate flow rates. At each of these flow rates, the probe would be driven across the channel at six different locations in the flow. These locations went from a position 0.100 inches from the convex wall surface to a distance of 0.025 inches from the surface of the concave wall. It was observed that when the assembly was driven in the transverse direction and then reversed at the same micrometer setting and flow rate that the result was a velocity pattern of the same magnitude and characteristics but slightly displaced in location. After close investigation it was determined that this variation was caused by the torque placed on the probe by the traversing assembly. In order that a detailed comparison could be made between the velocity profiles at various flow rates, it was determined to take all data with the sweep proceeding in one direction only.

For each run at a particular flow rate, the blower speed was adjusted and the equipment was allowed to stabilize for approximately thirty minutes before data was taken. The temperature of the entering fluid was taken, by mercury thermometer, every hour as the experiment proceeded. The flow rates corresponding to Reynolds numbers of 811, 1060, 1269, and 1518 were examined in detail.

The next area of investigation was to examine the lower flow rates and investigate the critical Reynolds number at which Taylor-Goertler vortices become apparent. This was accomplished by adjusting the flow rate to the smallest value

measurable on the rotometer and setting the probe to a micrometer adjustment of 0.025 inches away from the concave wall. Investigation was started at a Reynolds number of 209. The flow rate was increased in small increments until the characteristics of the vortices became apparent.

The final area of investigation was to determine the velocity profile of the vortices in the radial direction. This was accomplished by locating the probe along the transverse direction until a velocity maximum was indicated while the flow rate was maintained at a Reynolds number of 1047. This flow rate was selected because it provided large, distinct velocity fluctuations. At this location the transverse drive was stopped and the probe was moved radially from 0.100 inches from the convex wall to 0.010 inches from the concave wall in increments of 0.020 inches. The horizontal scale of the X-Y recorder was changed to indicate the depth of the probe into the channel. After this profile had been completed, the probe was again moved in the transverse direction until a velocity minimum was encountered. At this point the traversing assembly was again stopped and the procedure described previously was repeated.

B. DISCUSSION OF RESULTS

The velocity measured by the probe was only that existing in a plane perpendicular to the measuring wire. Because of this limitation, the velocity profile recorded and shown in the figures which follow was the vector sum of the radial and circumferential velocity. The Reynolds number for the

following figures was defined using the mean velocity indicated by the rotometer and hydraulic diameter of 0.04065 feet.

Figures 8 through 13 show the measured velocity at a Reynolds number of 1060. It was observed that as the distance from the concave wall increased the magnitude of the velocity fluctuations decreased. It was also apparent that the vortices were uniform and evenly spaced, as would have been concluded from the work of Goertler. Figures 14 through 19, Reynolds number of 1518, apparently demonstrated the onset of turbulence, but it was also noted the periodic characteristics of the vortices persisted. These figures indicated that the magnitude of turbulent fluctuation was small in the region of velocity maximums when compared to the magnitude in the center region of the vortex. It was also apparent that turbulent fluctuations decreased as the sensor moved toward the center of the channel. Figures 20 through 25, Reynolds number of 1269, are similar to those of 8 through 13 except for the increase in the magnitude of the velocity fluctuation and mean velocity. This result was expected with the increased flow rate. There was also an increase in the strength of the second vortex from the left in comparison to the others present. Figures 26 through 31 demonstrated the velocity profile at a Reynolds number of 811. From these results it was evident that the periodic variation was very slight even though the flow rate was well above the flow rate at which the vortices should have formed. Figures 32 and 34, Reynolds number of 1047, demonstrated that the mean velocity maximum did not

occur at the center of the channel but was displaced in the direction of centrifugal force, toward the concave wall. This result was in agreement with the findings of McKee and also those of Mori, Uchida, and Ukon in their investigation of flow in curved channels of square cross section. It was also noted that this shift was not apparent at the vortex center, demonstrated by figure 33. It should be pointed out that figures 32, 33, and 34 represent a radial velocity profile at vortex edge, vortex center, and vortex edge respectively. Figure 35 demonstrated that, as the flow rate was increased from a Reynolds number of 209 to 340 the velocity apparently increased in that portion of the observation area closest to the center of the channel. The critical value of Dean number is 36, and based on this the critical value of Reynolds number is 245. The distinct periodic variations were not observable until the Reynolds number was in excess of 700.

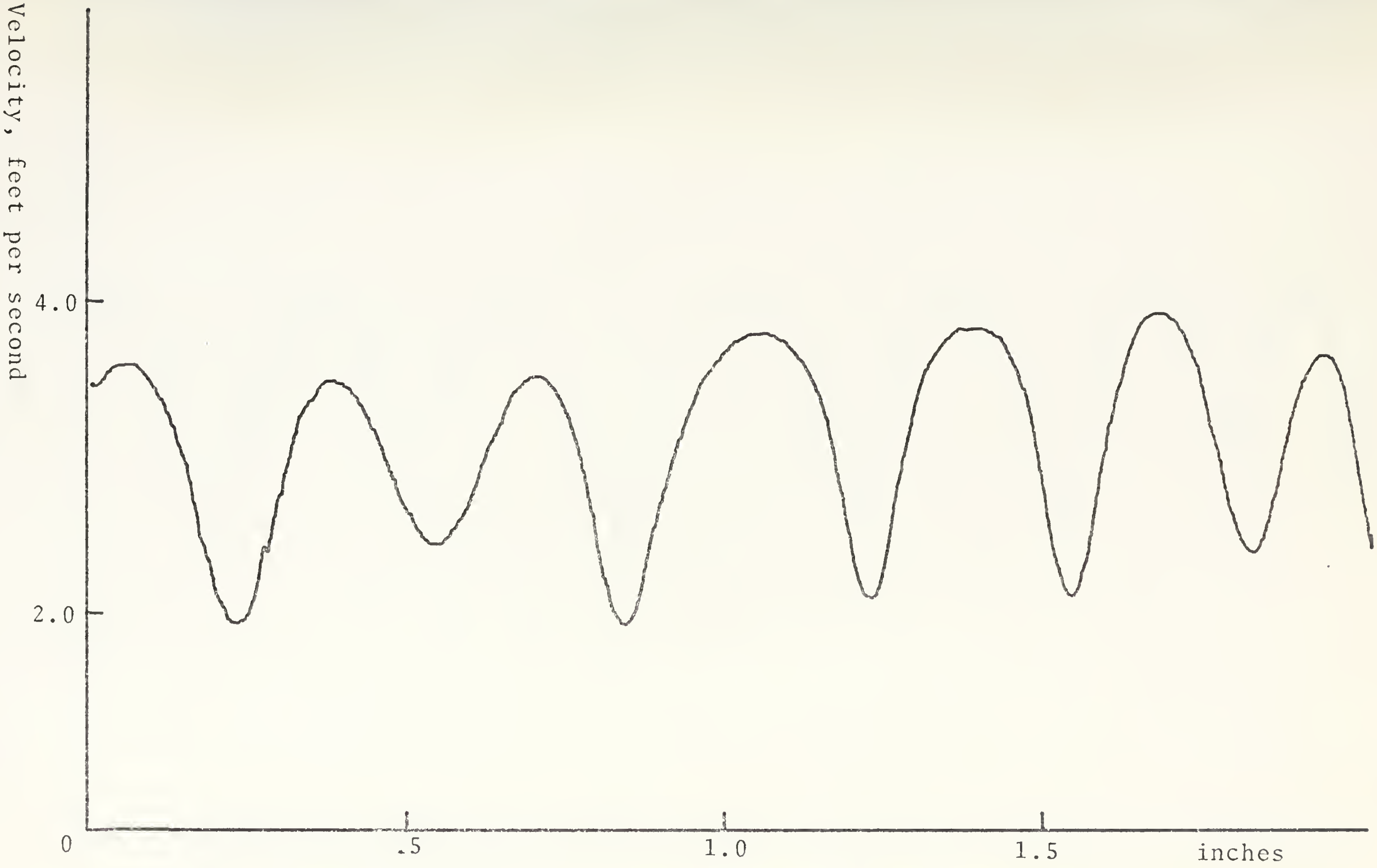

FIGURE 8. Velocity Profile R_e = 1060, Distance from Concave Wall = 0.025 inches.

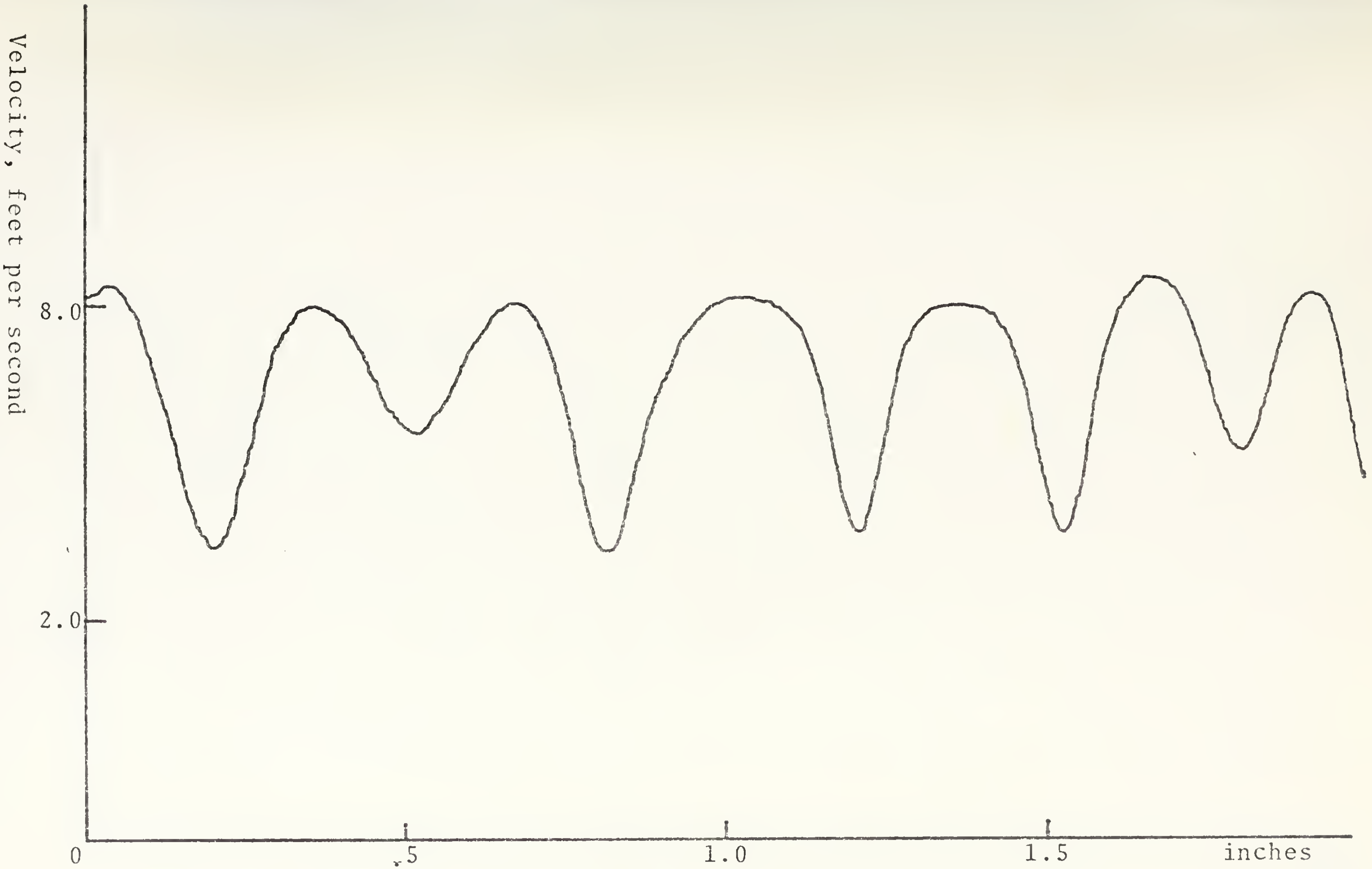

FIGURE 9. Velocity Profile R_e = 1060, Distance from Concave Wall = 0.050 inches.

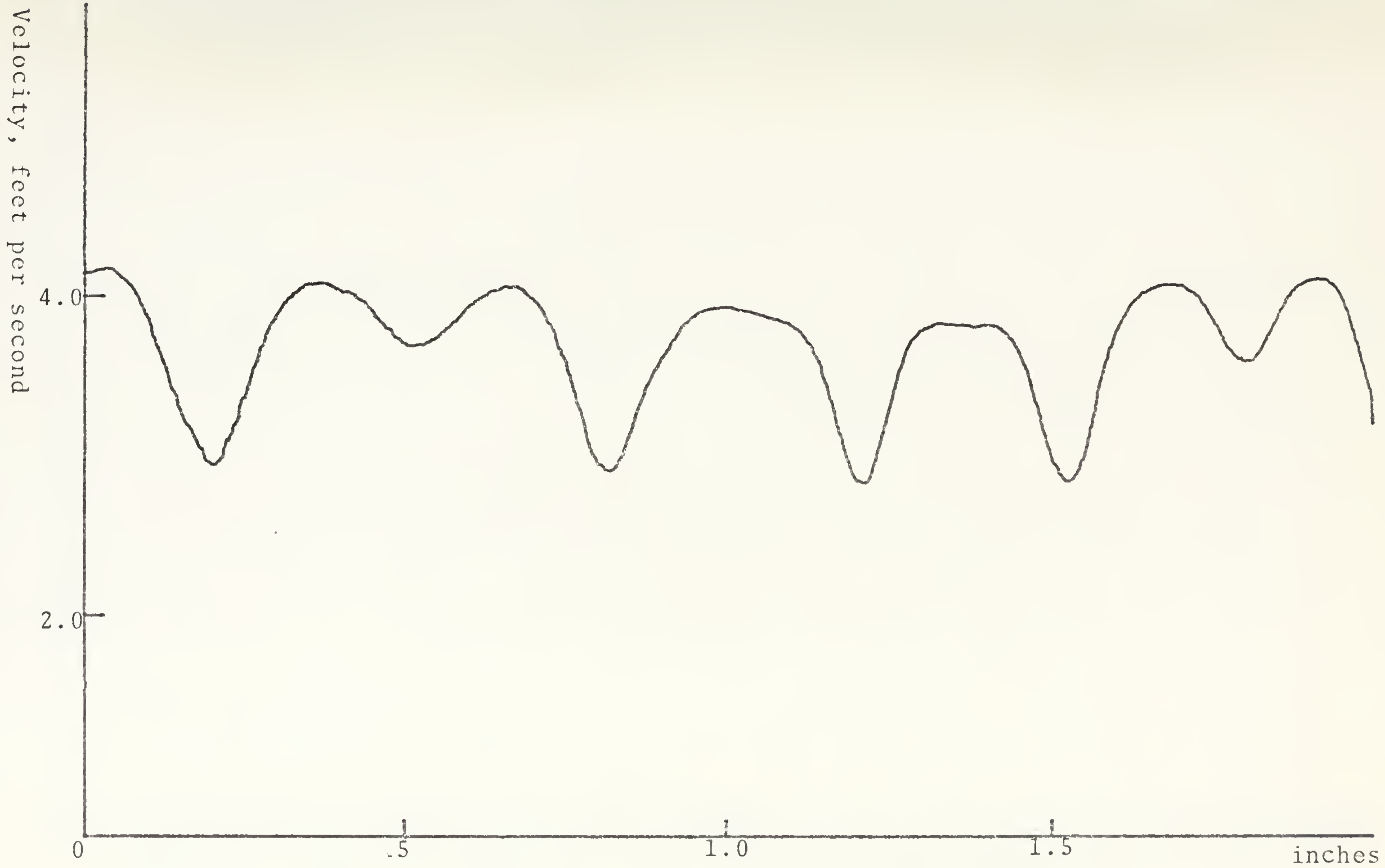

FIGURE 10. Velocity Profile R_e = 1060, Distance from Concave Wall = 0.075 inches.

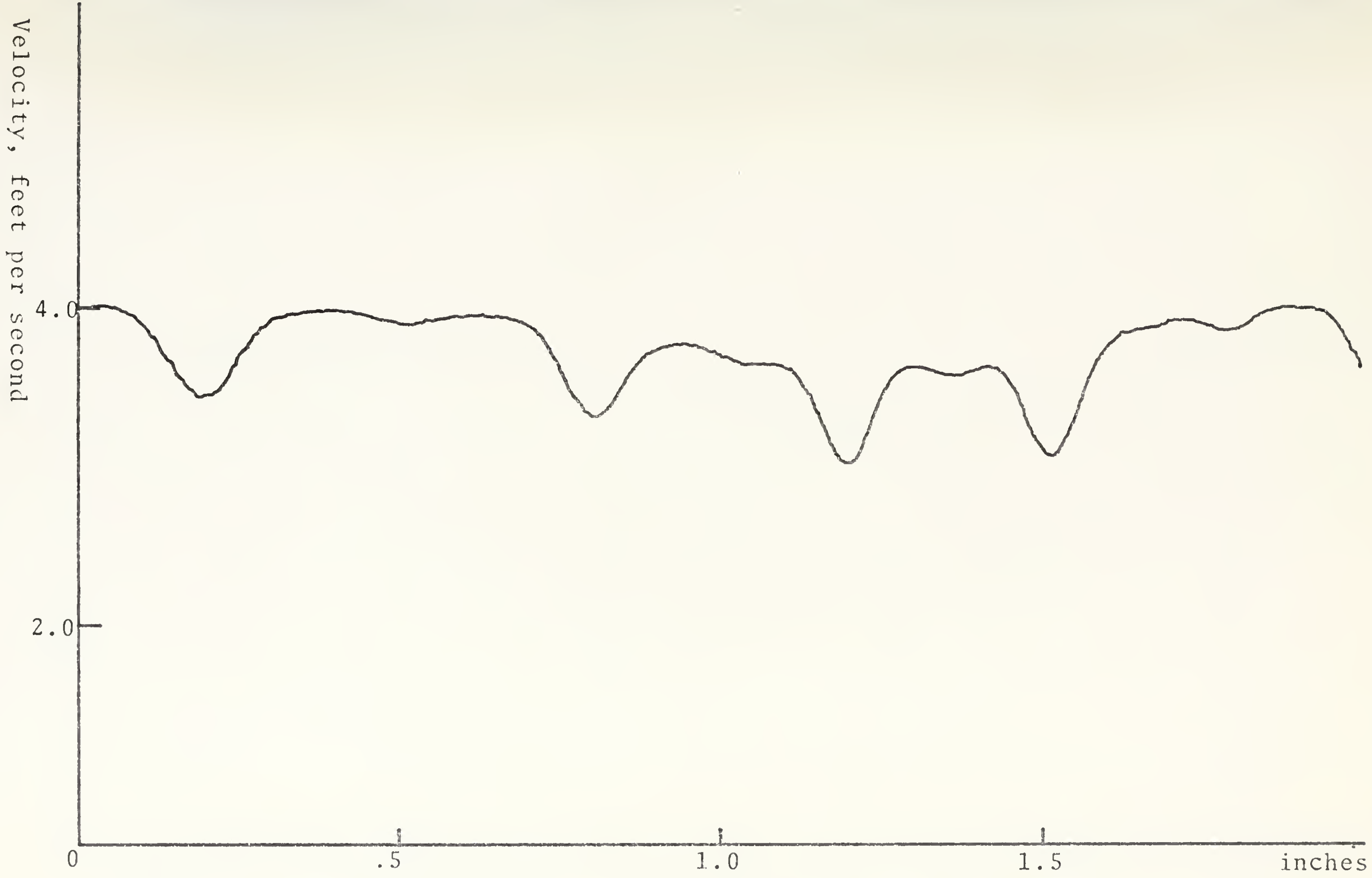

FIGURE 11. Velocity Profile R_e = 1060, Distance from Concave Wall = 0.100 inches.

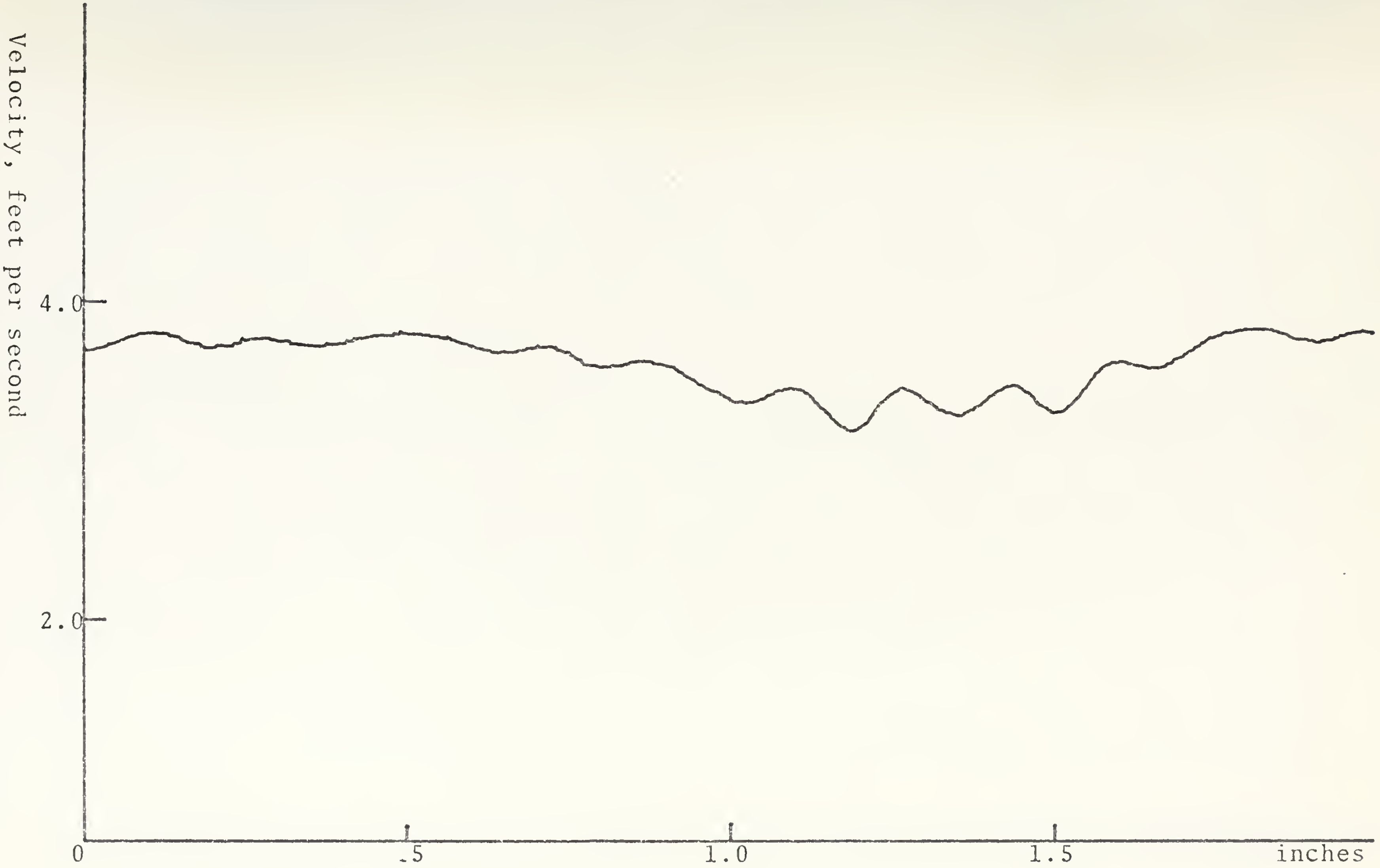

FIGURE 12. Velocity Profile R_e = 1060, Distance from Concave Wall = 0.125 inches.

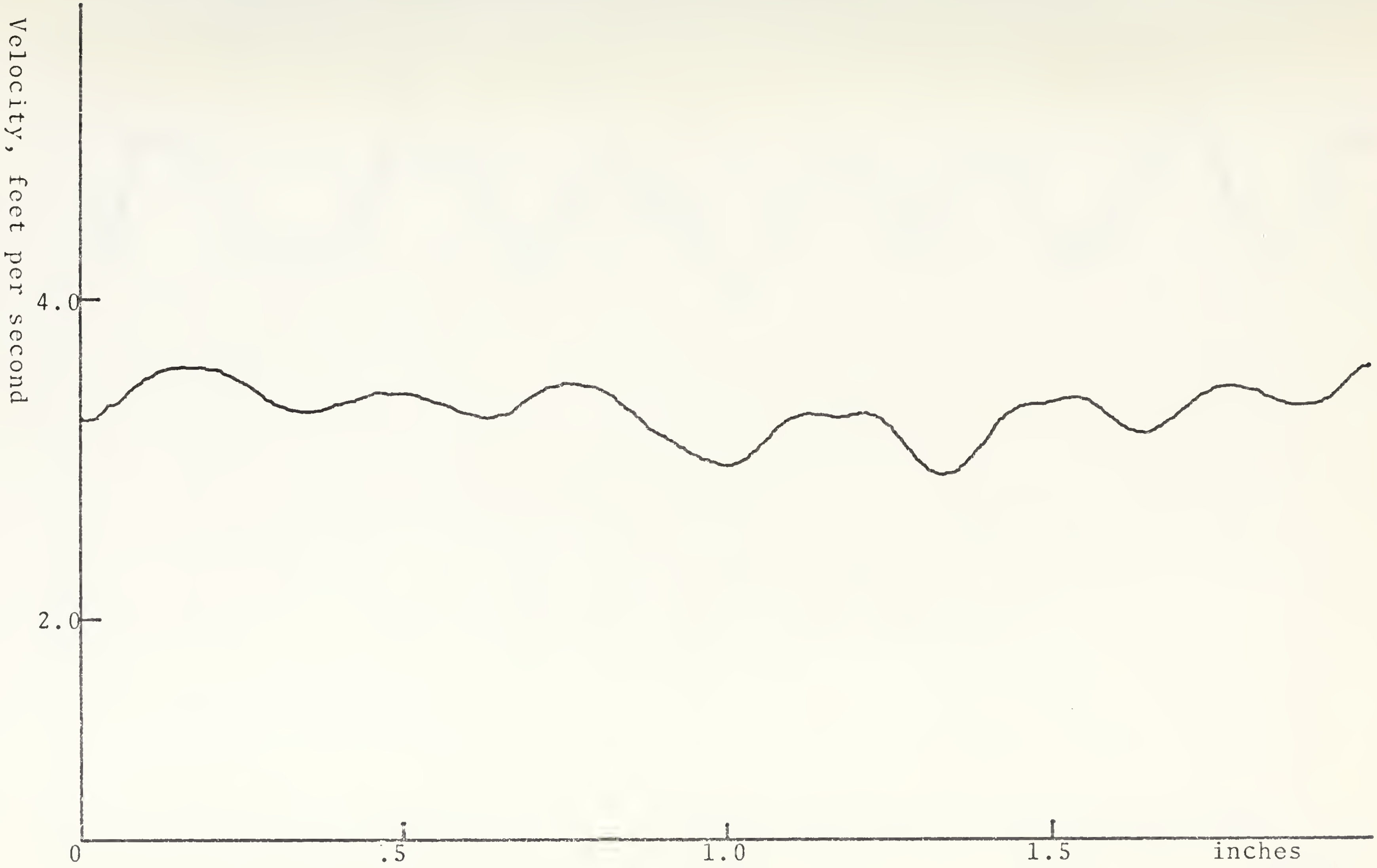

FIGURE 13. Velocity Profile R_e = 1060, Distance from Concave Wall = 0.150 inches.

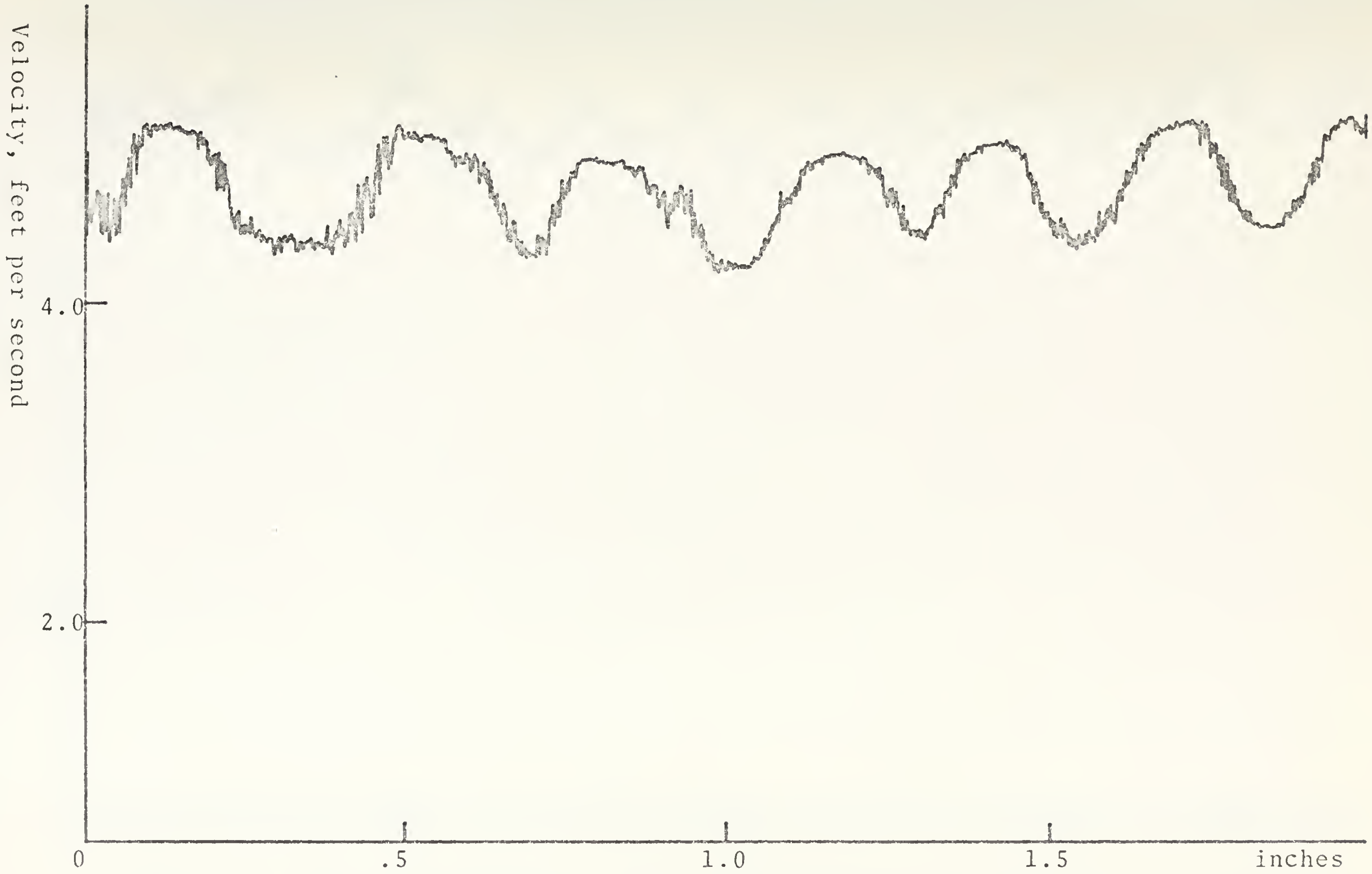

FIGURE 14. Velocity Profile R_e = 1518, Distance from Concave Wall = 0.150 inches.

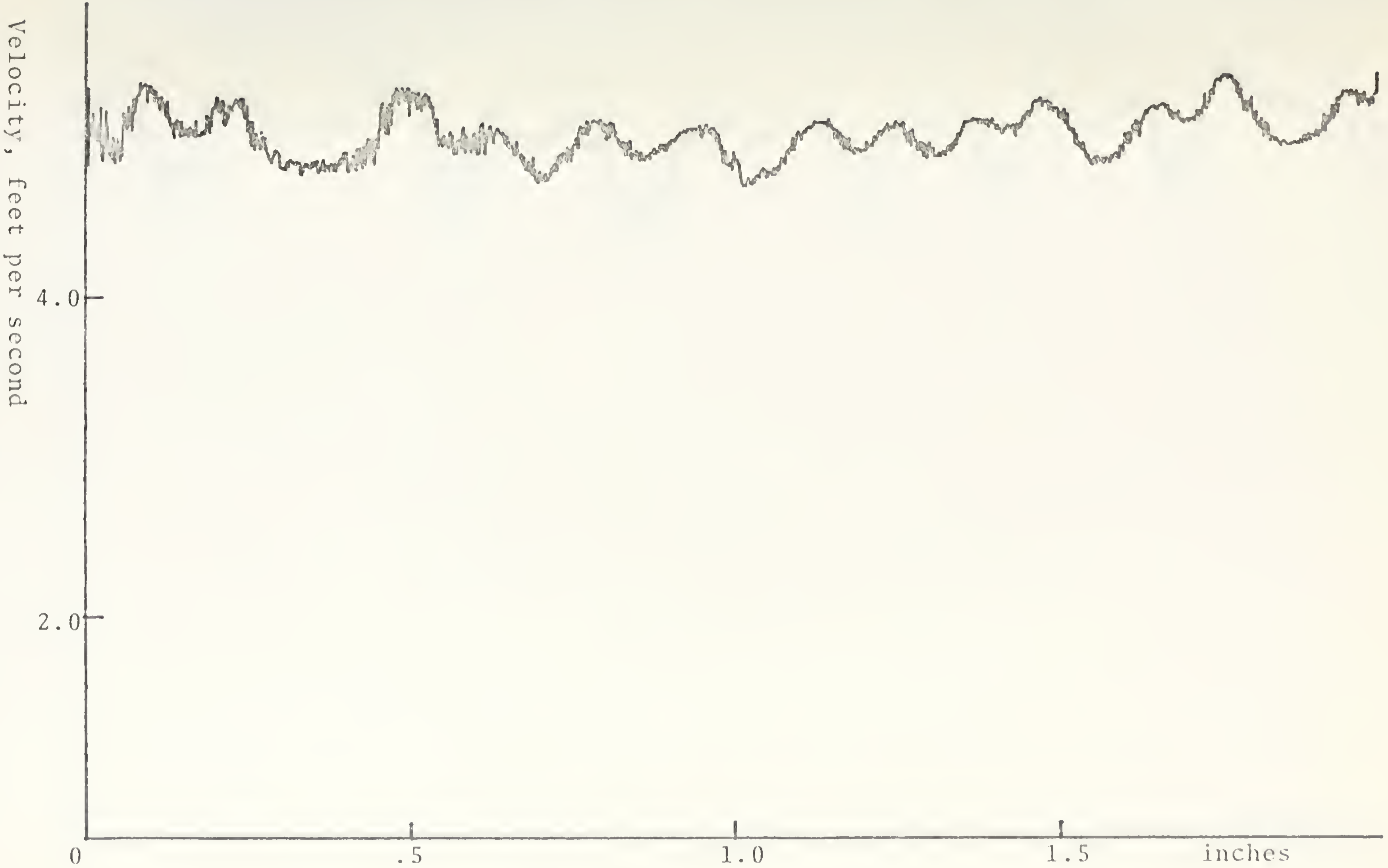

FIGURE 15. Velocity Profile R_c = 1518, Distance from Concave Wall = 0.125 inches.

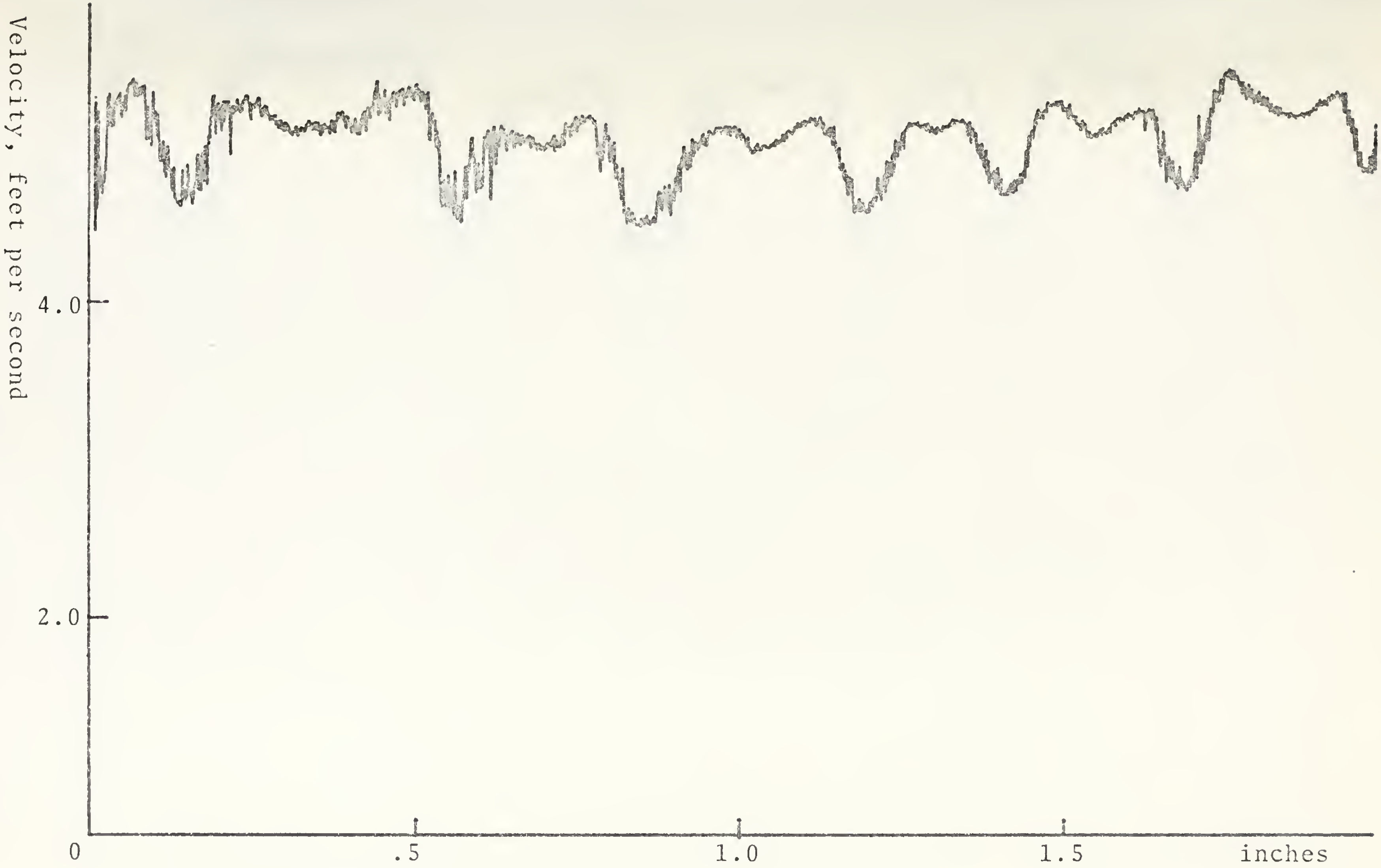

FIGURE 16. Velocity Profile R_e = 1518, Distance from Concave Wall = 0.100 inches.

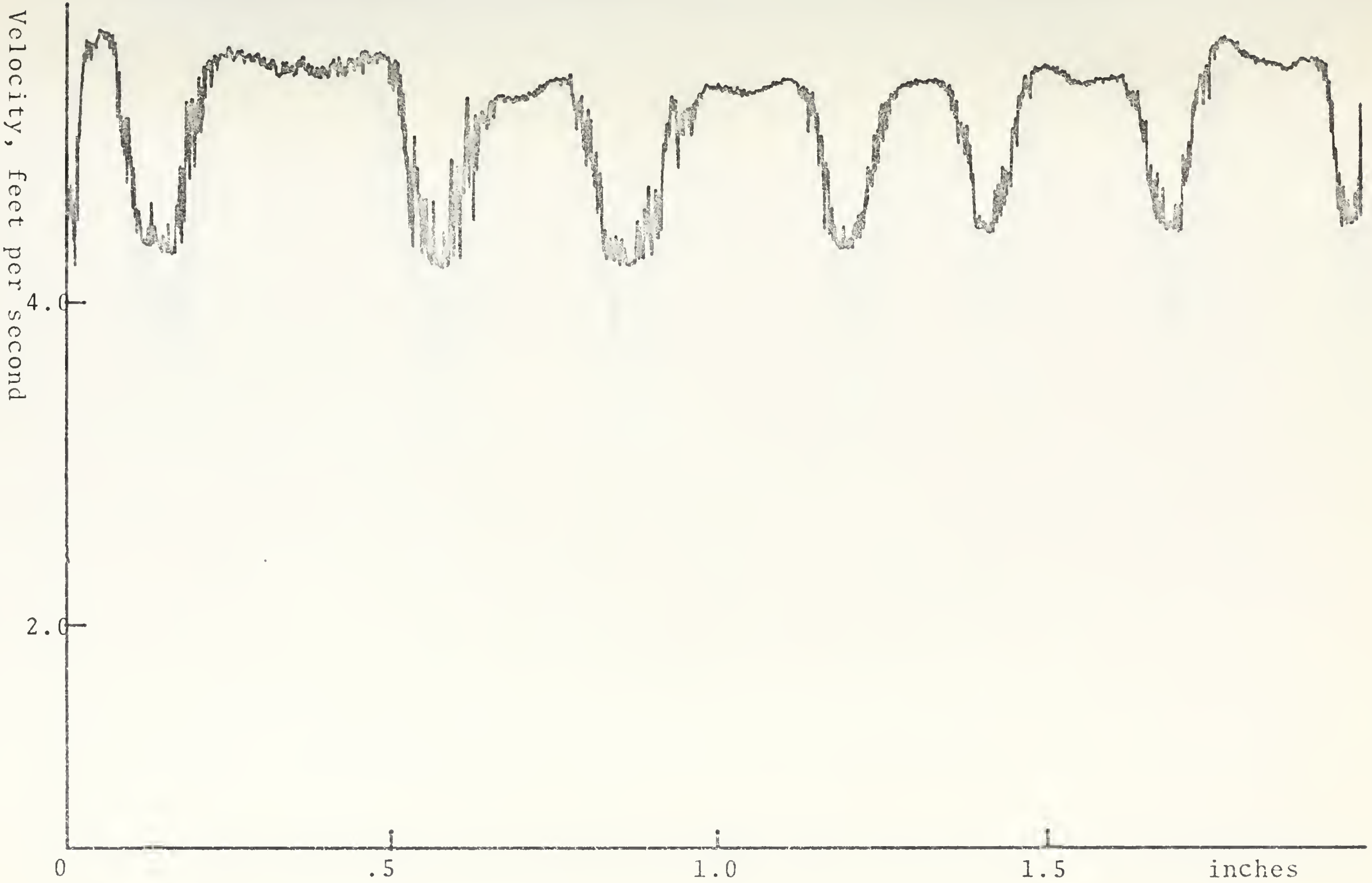

FIGURE 17. Velocity Profile R_e = 1518, Distance from Concave Wall = 0.075 inches.

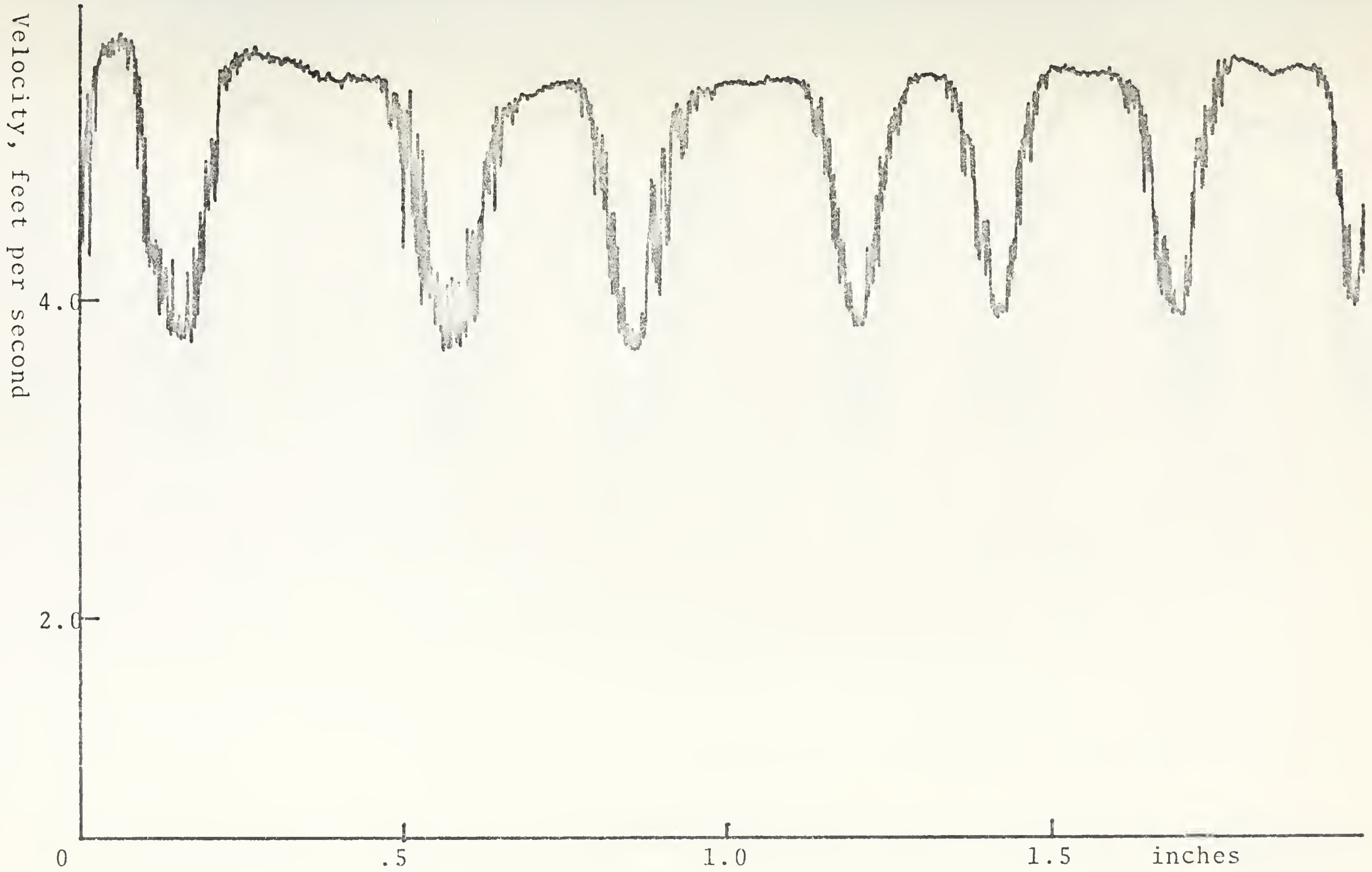

FIGURE 18. Velocity Profile R_e = 1518, Distance from Concave Wall = 0.050 inches.

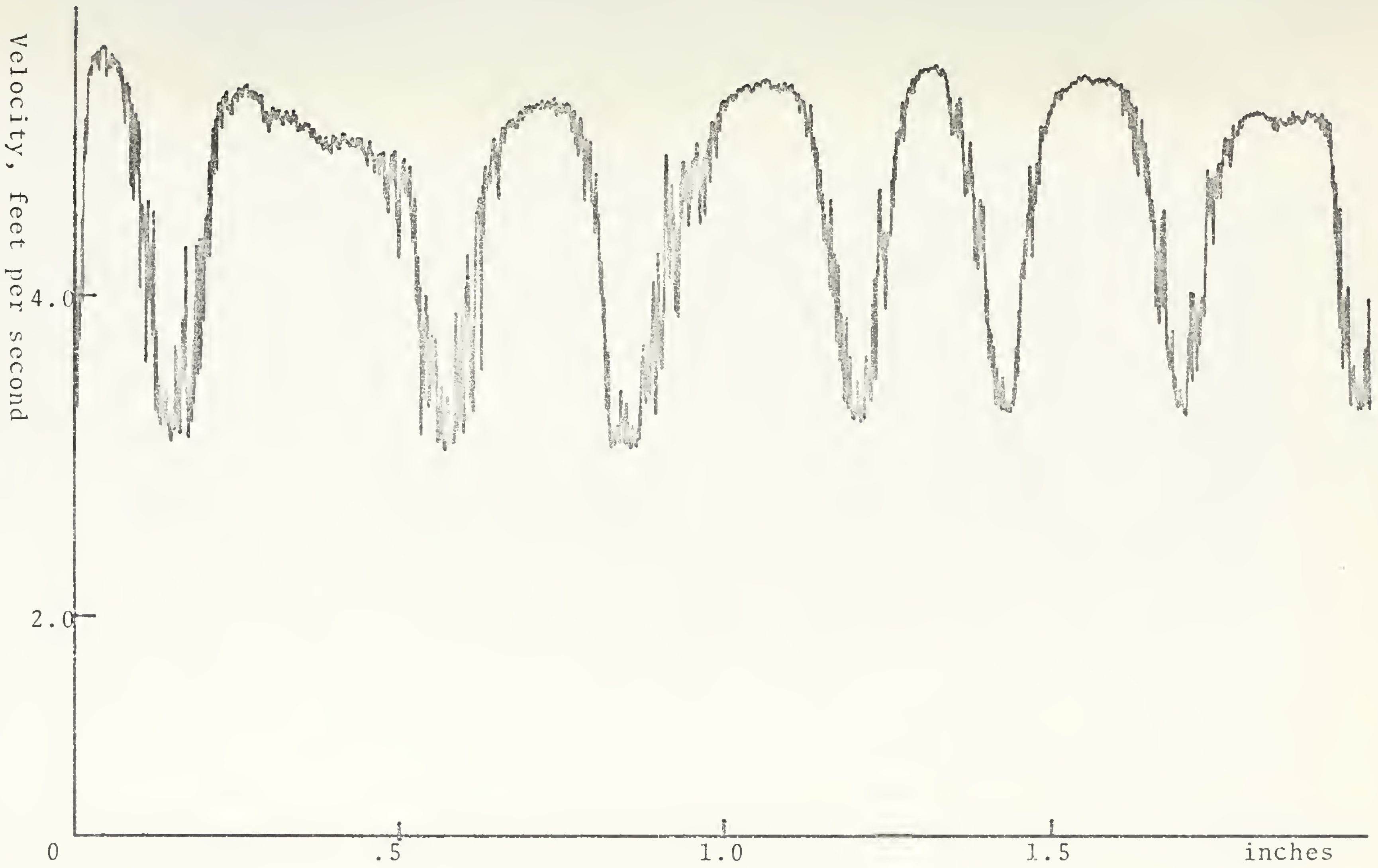

FIGURE 19. Velocity Profile R_e = 1518, Distance from Concave Wall = 0.025 inches.

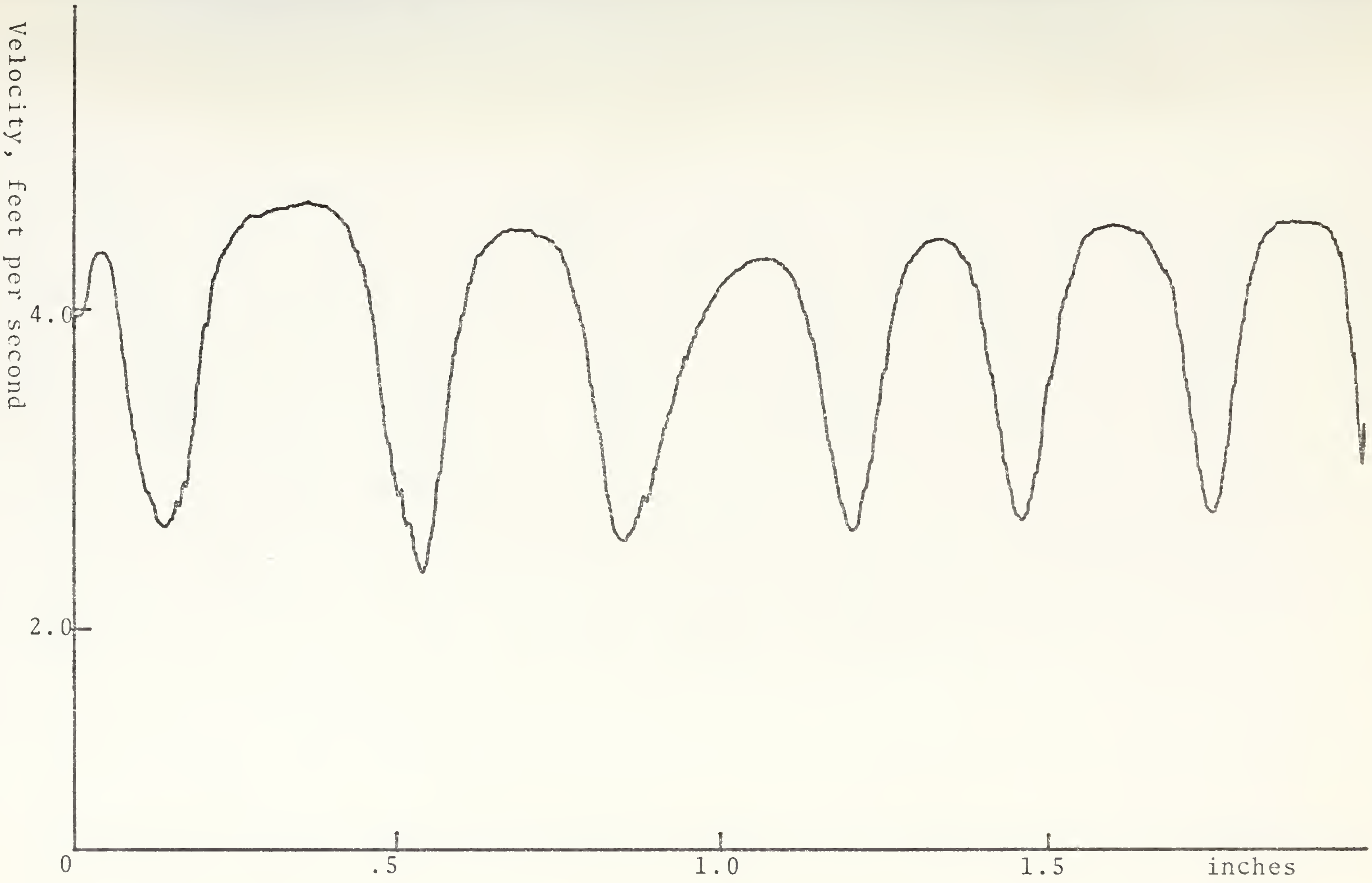

FIGURE 20. Velocity Profile R_e = 1269, Distance from Concave Wall = 0.025 inches.

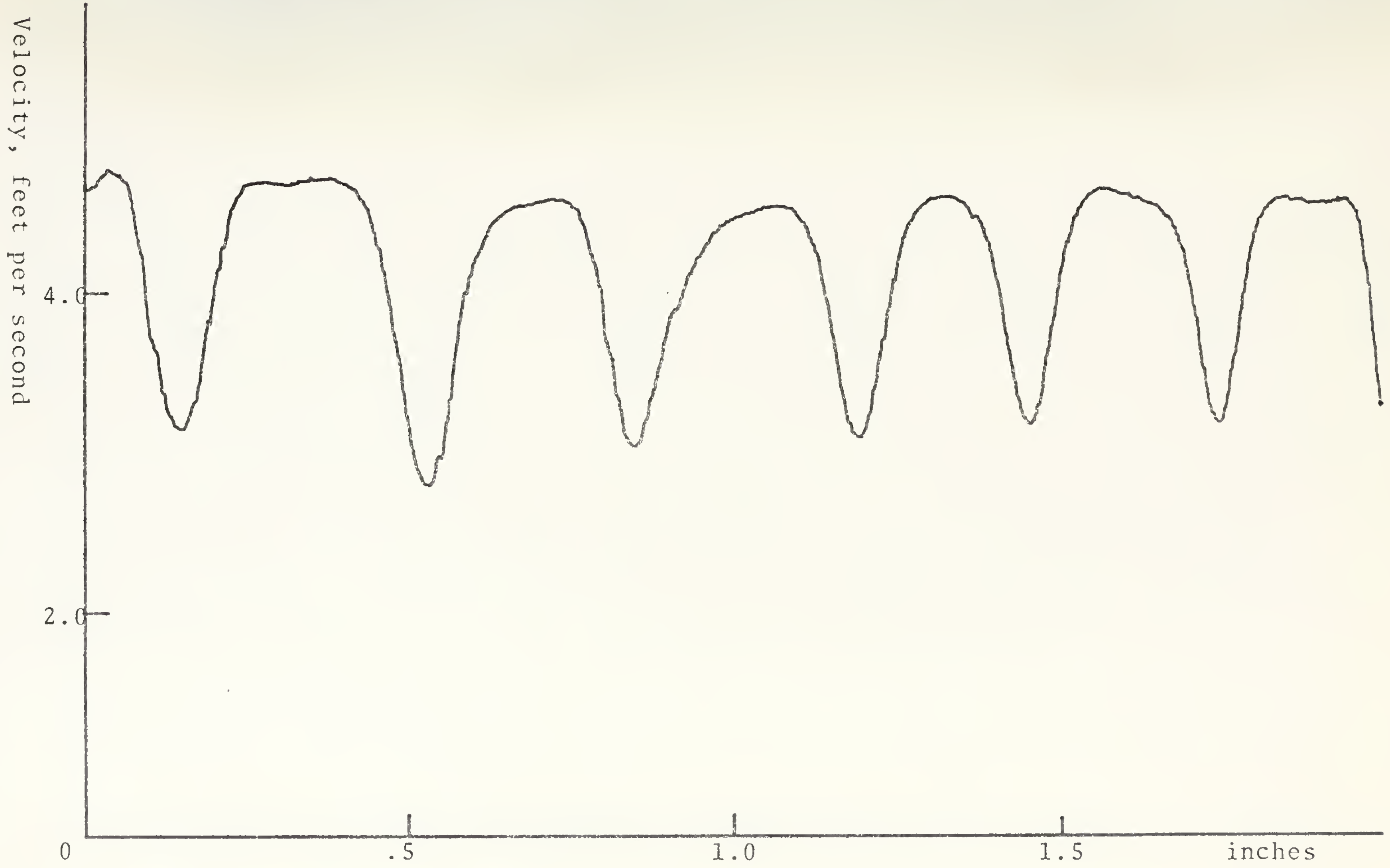

FIGURE 21. Velocity Profile R_e = 1269, Distance from Concave Wall = 0.050 inches.

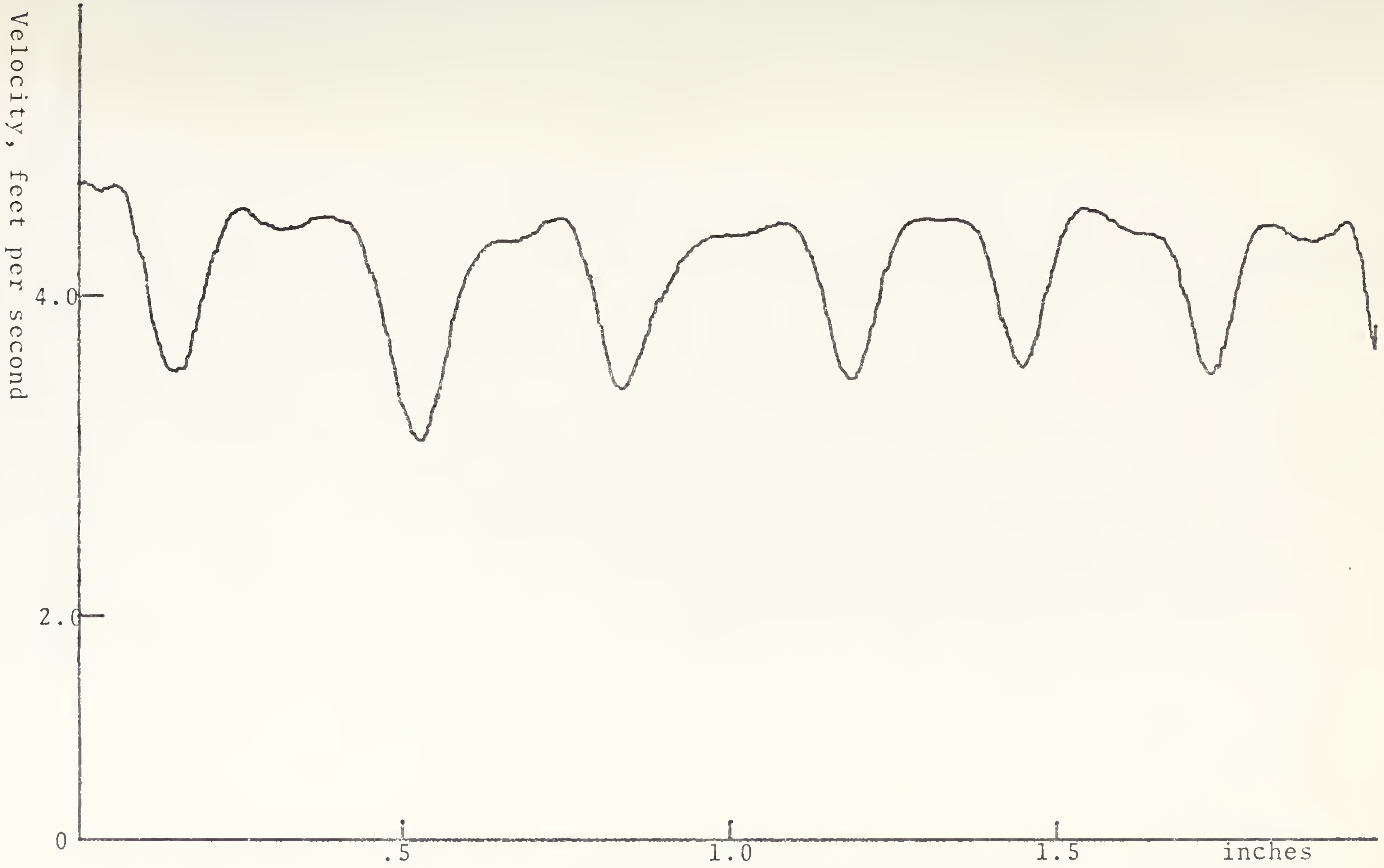

FIGURE 22. Velocity Profile R_e = 1269, Distance from Concave Wall = 0.075 inches.

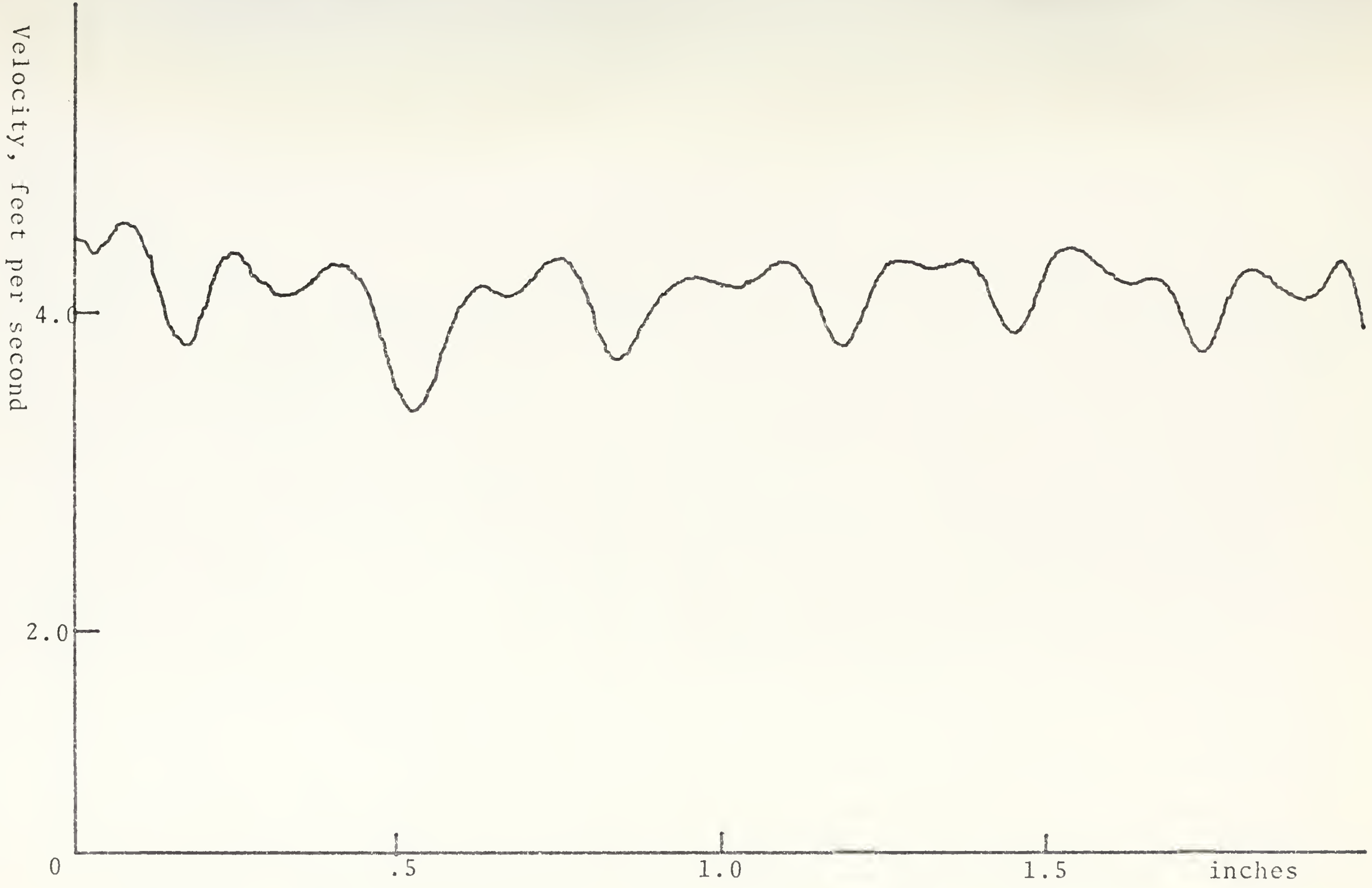

FIGURE 23. Velocity Profile $R_e = 1269$, Distance from Concave Wall = 0.100 inches.

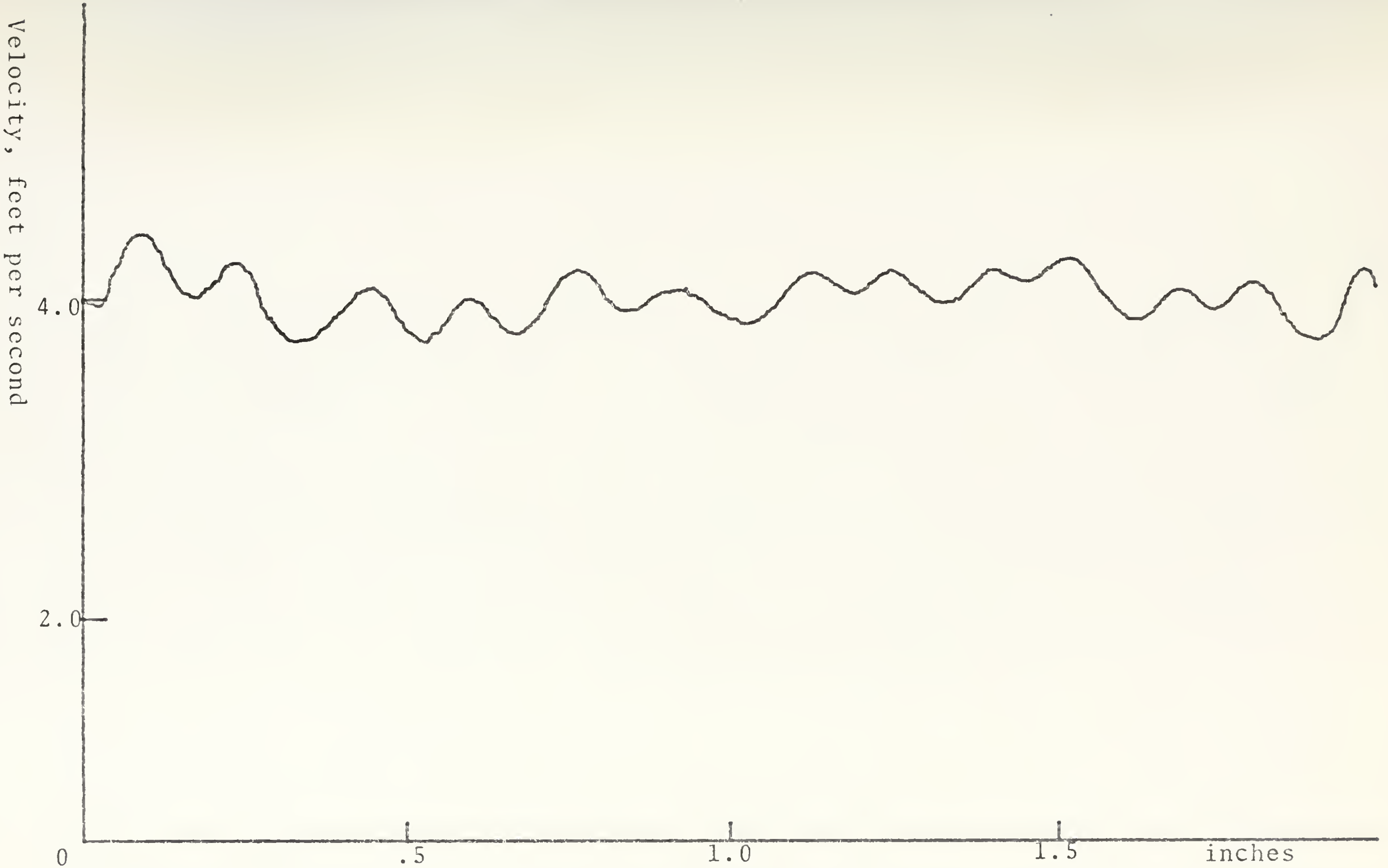

FIGURE 24. Velocity Profile R_e = 1269, Distance from Concave Wall = 0.125 inches.

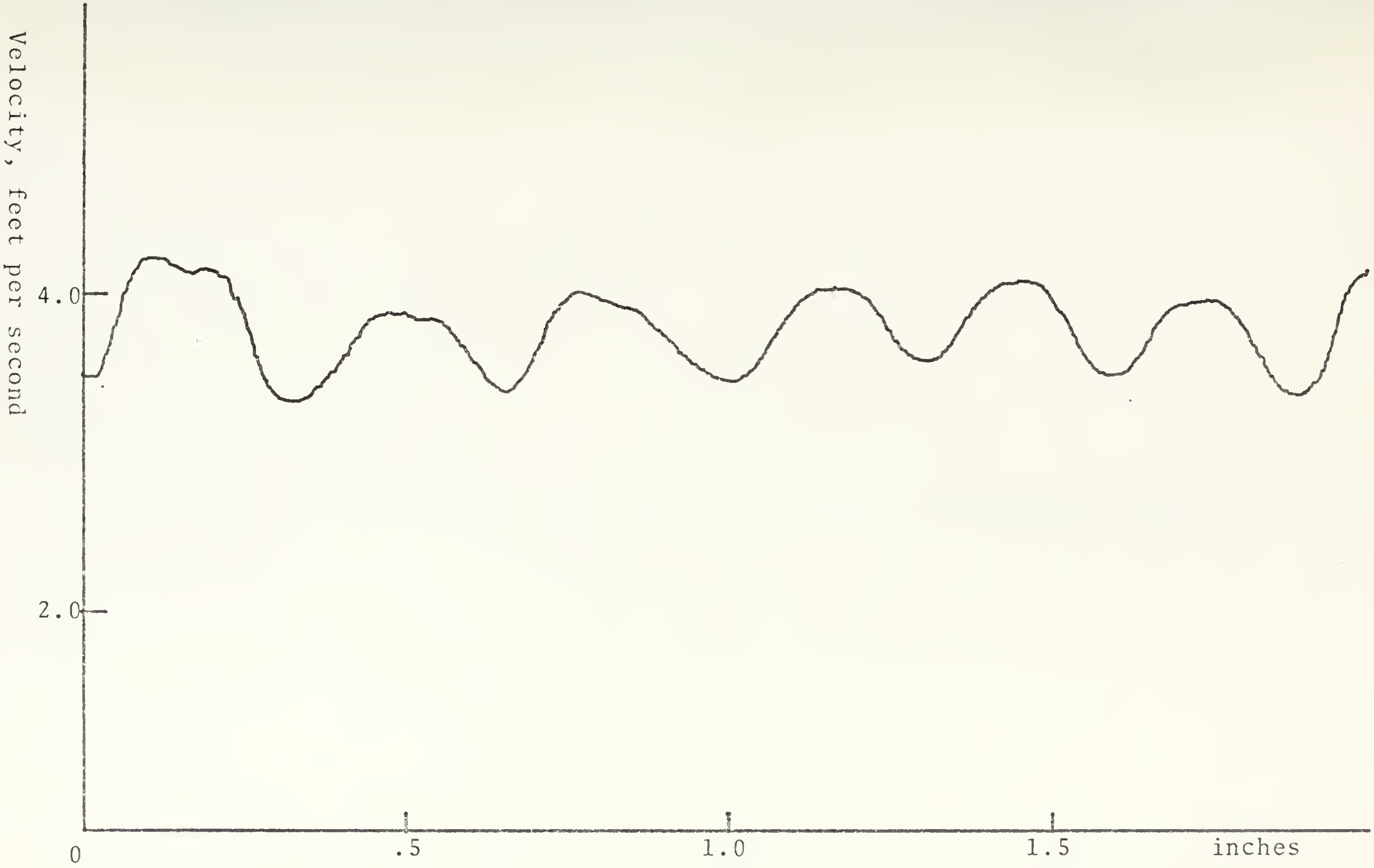

FIGURE 25. Velocity Profile R_e = 1269, Distance from Concave Wall = 0.150 inches.

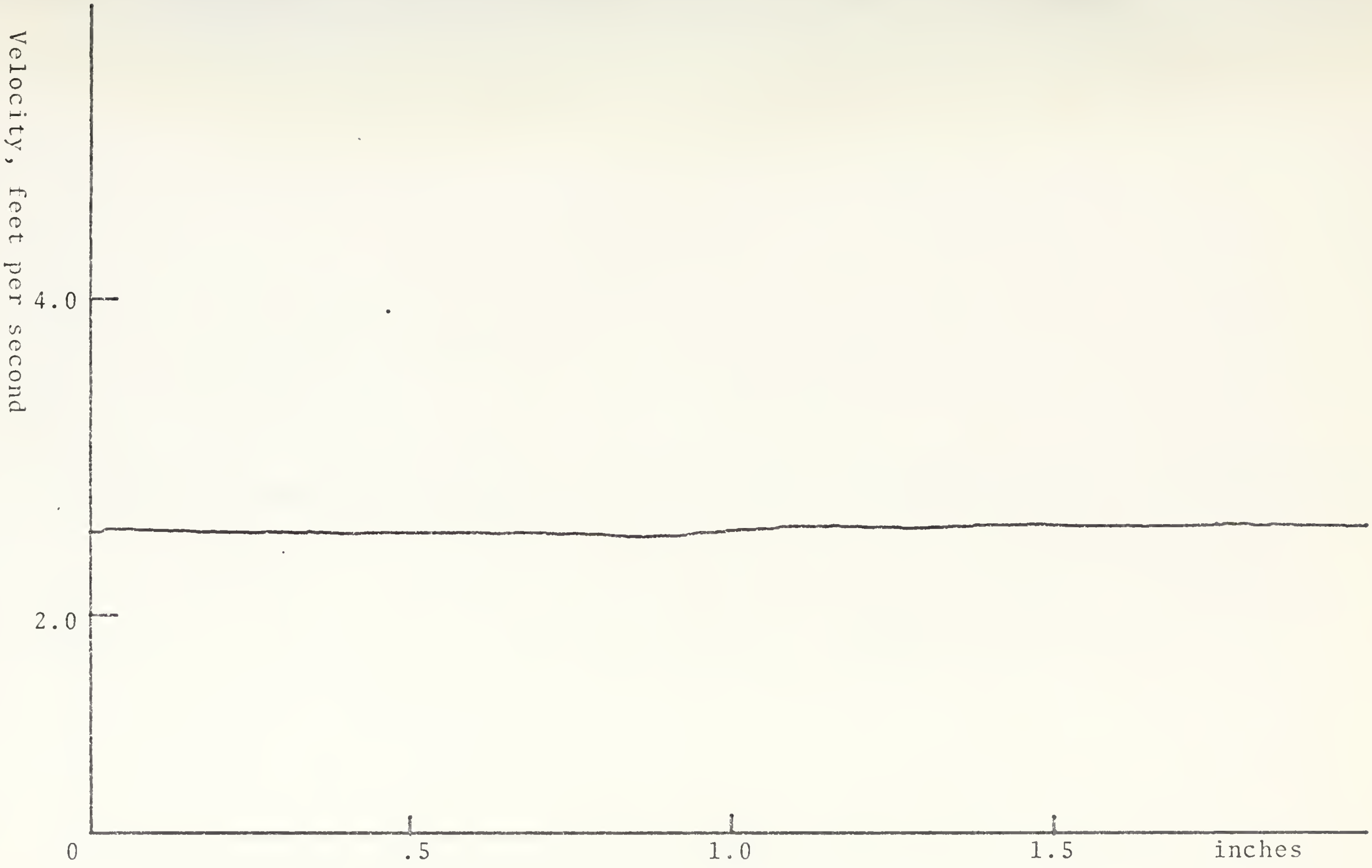

FIGURE 26. Velocity Profile R_e = 811, Distance from Concave Wall = 0.150 inches.

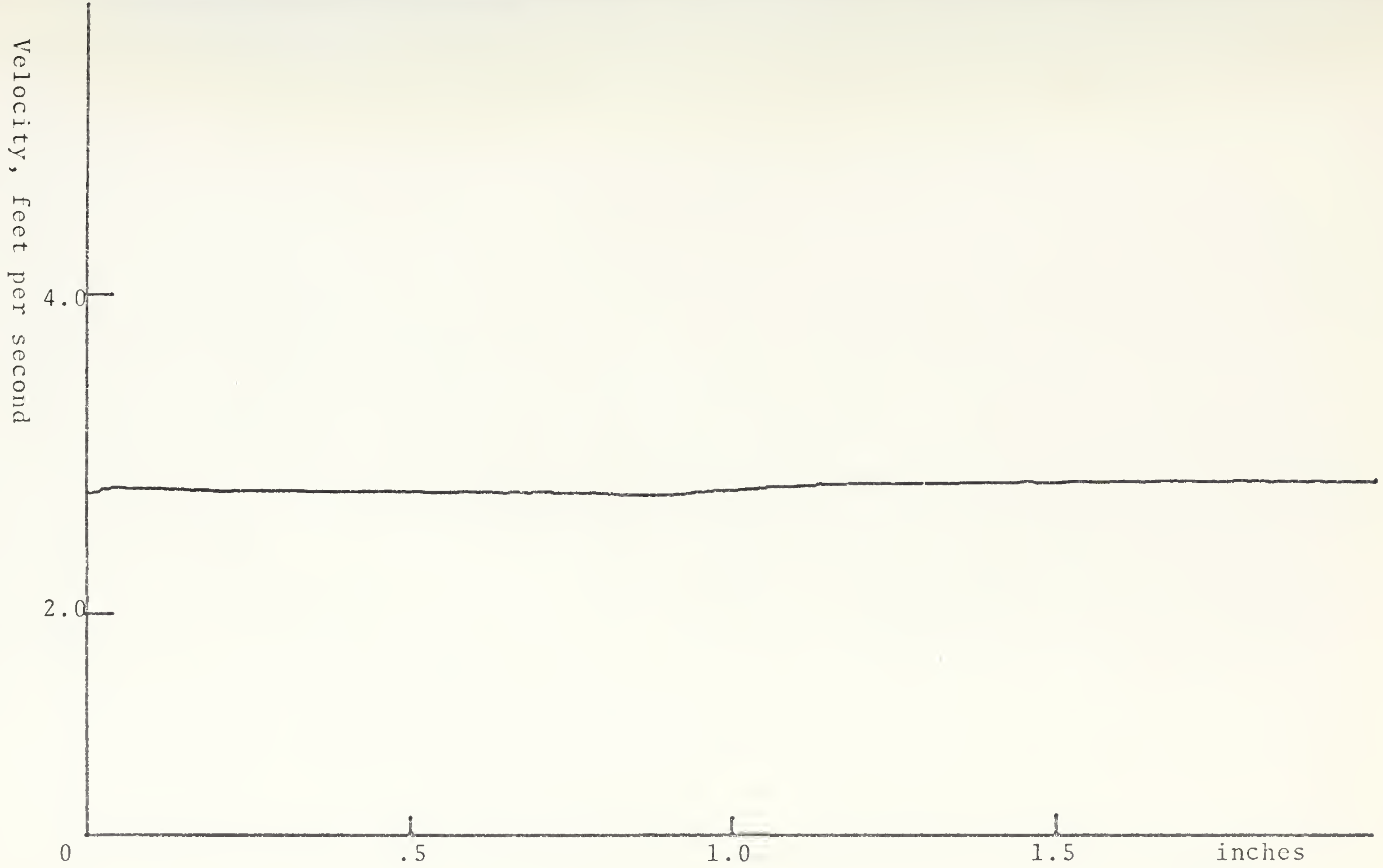

FIGURE 27. Velocity Profile R_e = 811, Distance from Concave Wall = 0.125 inches.

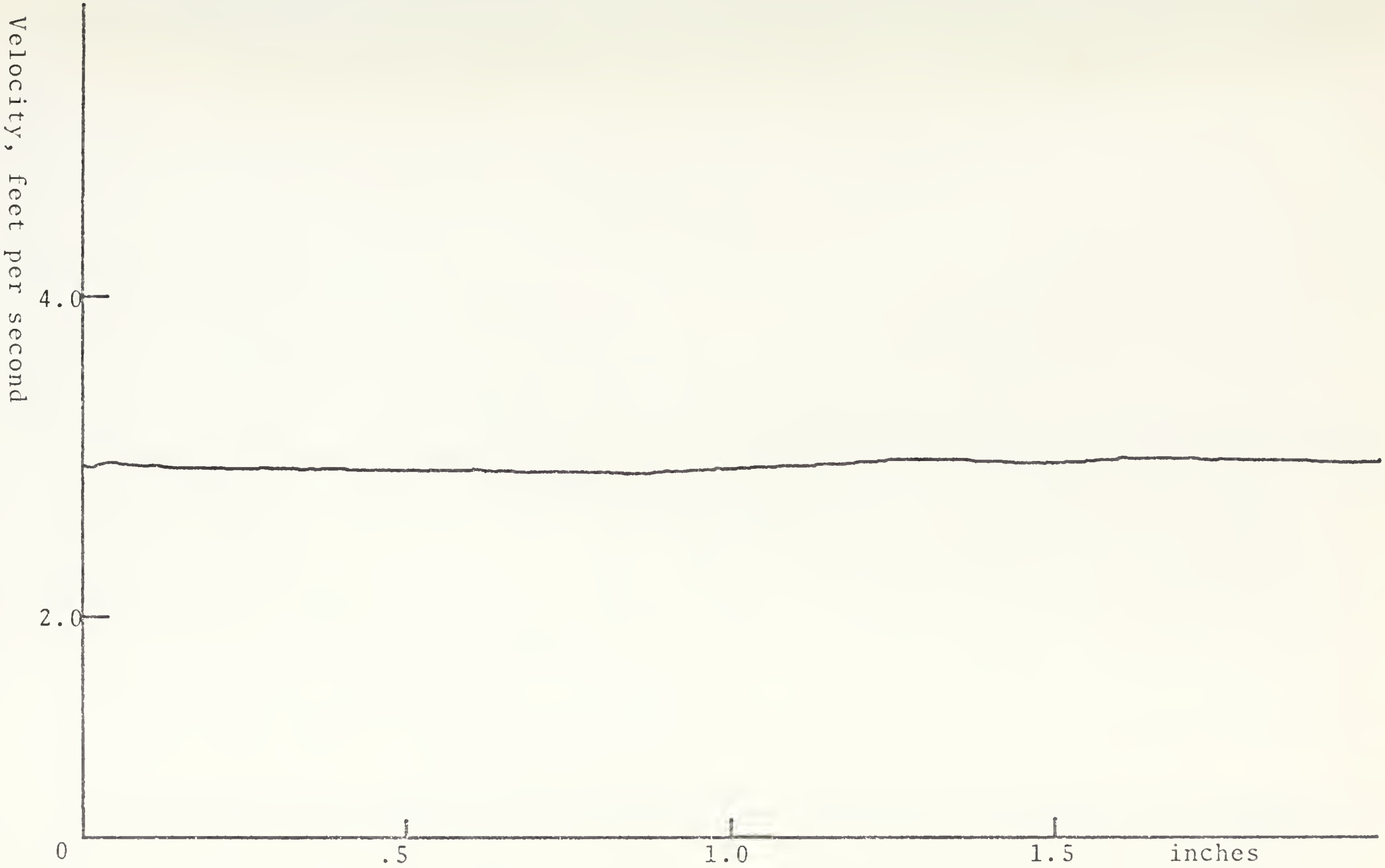

FIGURE 28. Velocity Profile R_e = 811, Distance from Concave Wall = 0.100 inches

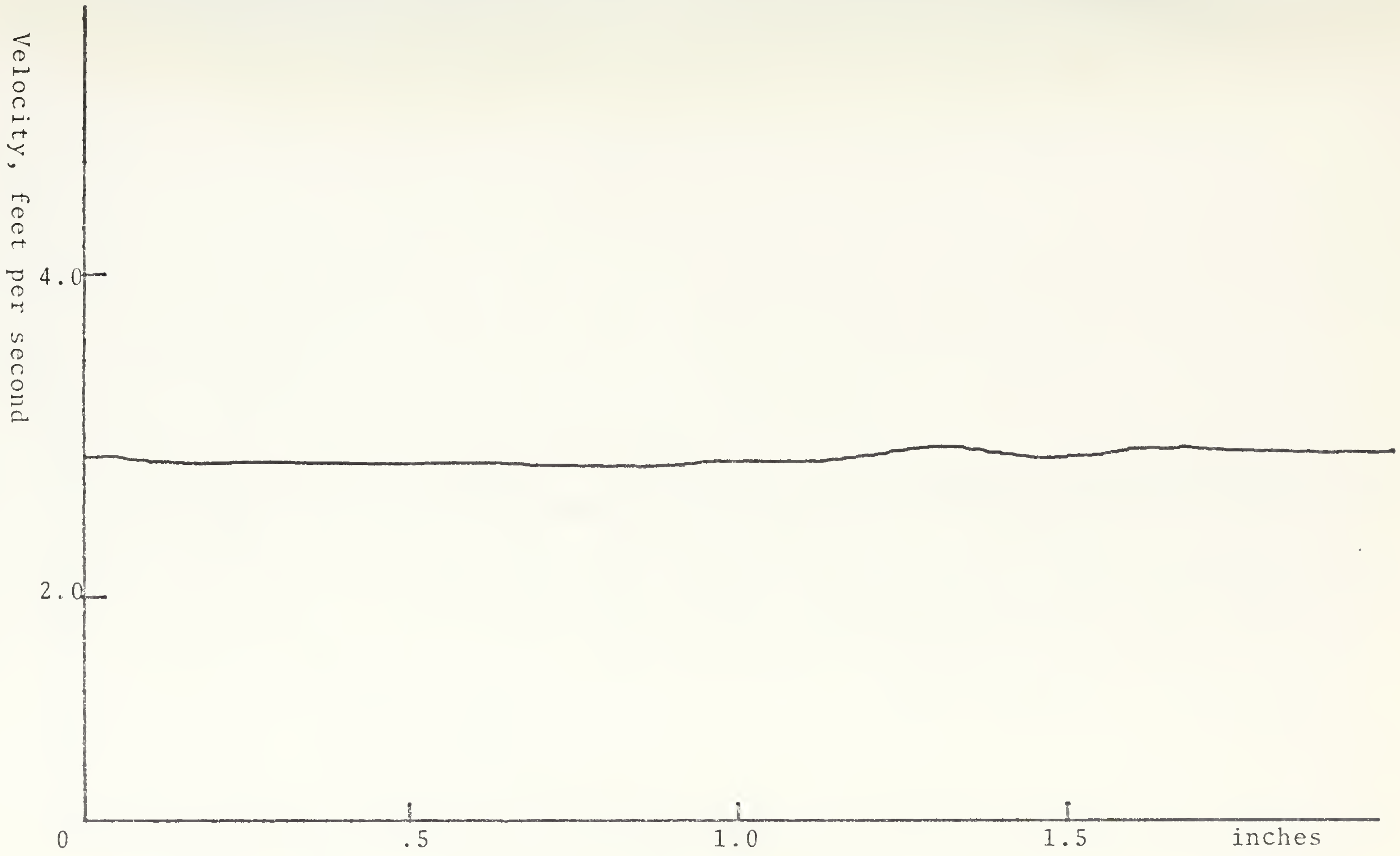

FIGURE 29. Velocity Profile R_e = 811, Distance from Concave Wall = 0.075 inches.

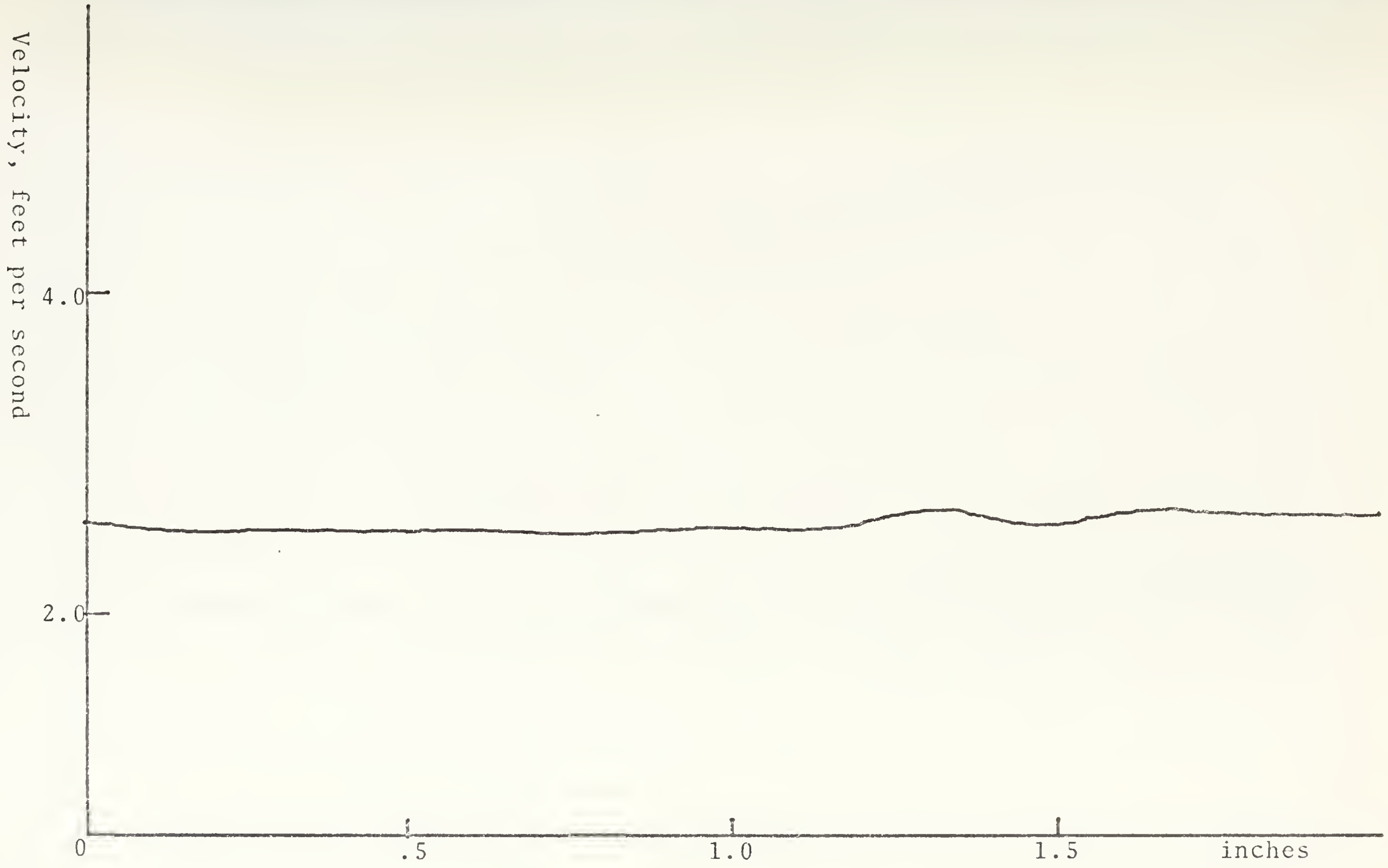

FIGURE 30. Velocity Profile R_e = 811, Distance from Concave Wall = 0.050 inches.

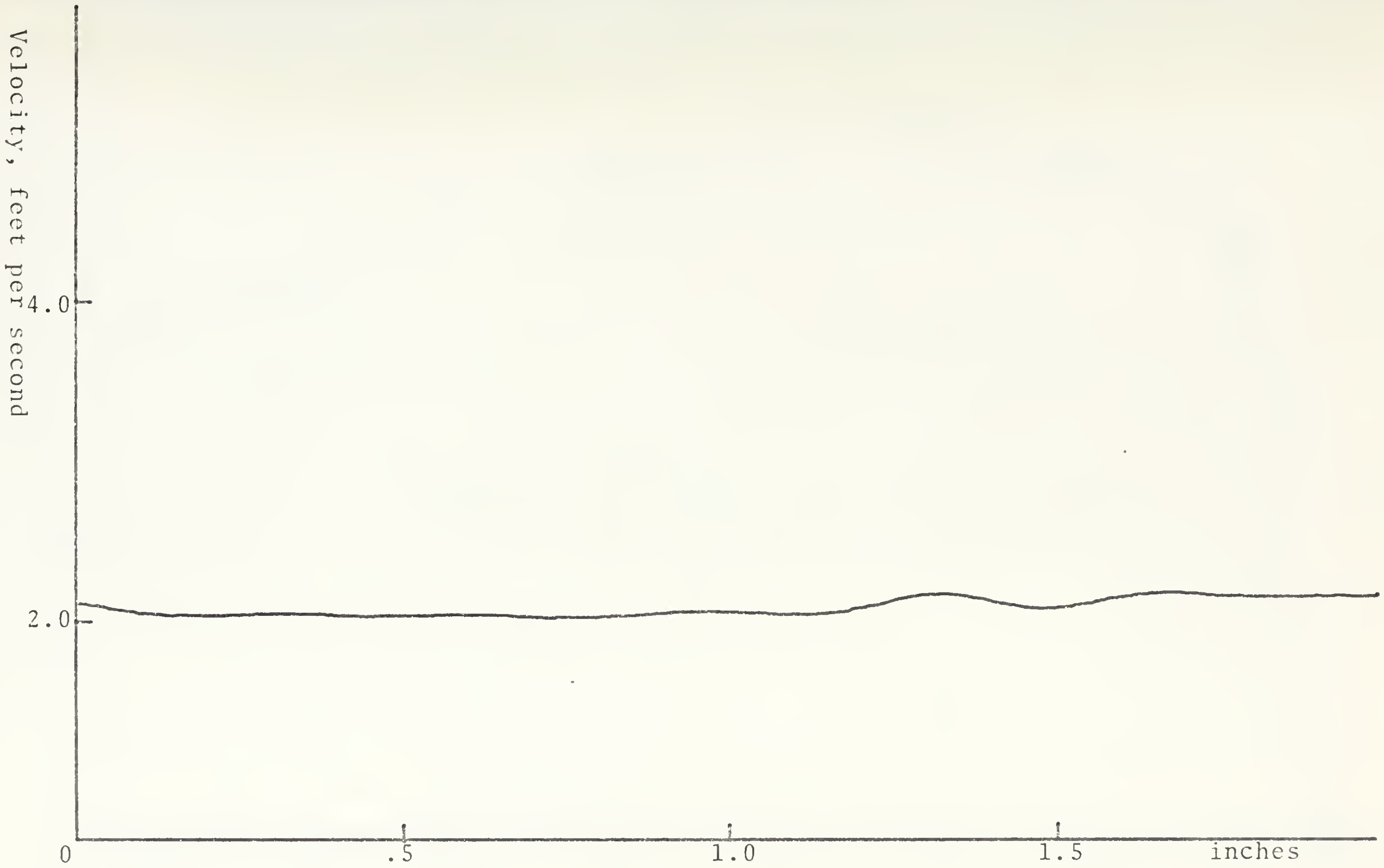

FIGURE 31. Velocity Profile R_e = 811, Distance from Concave Wall = 0.025 inches.

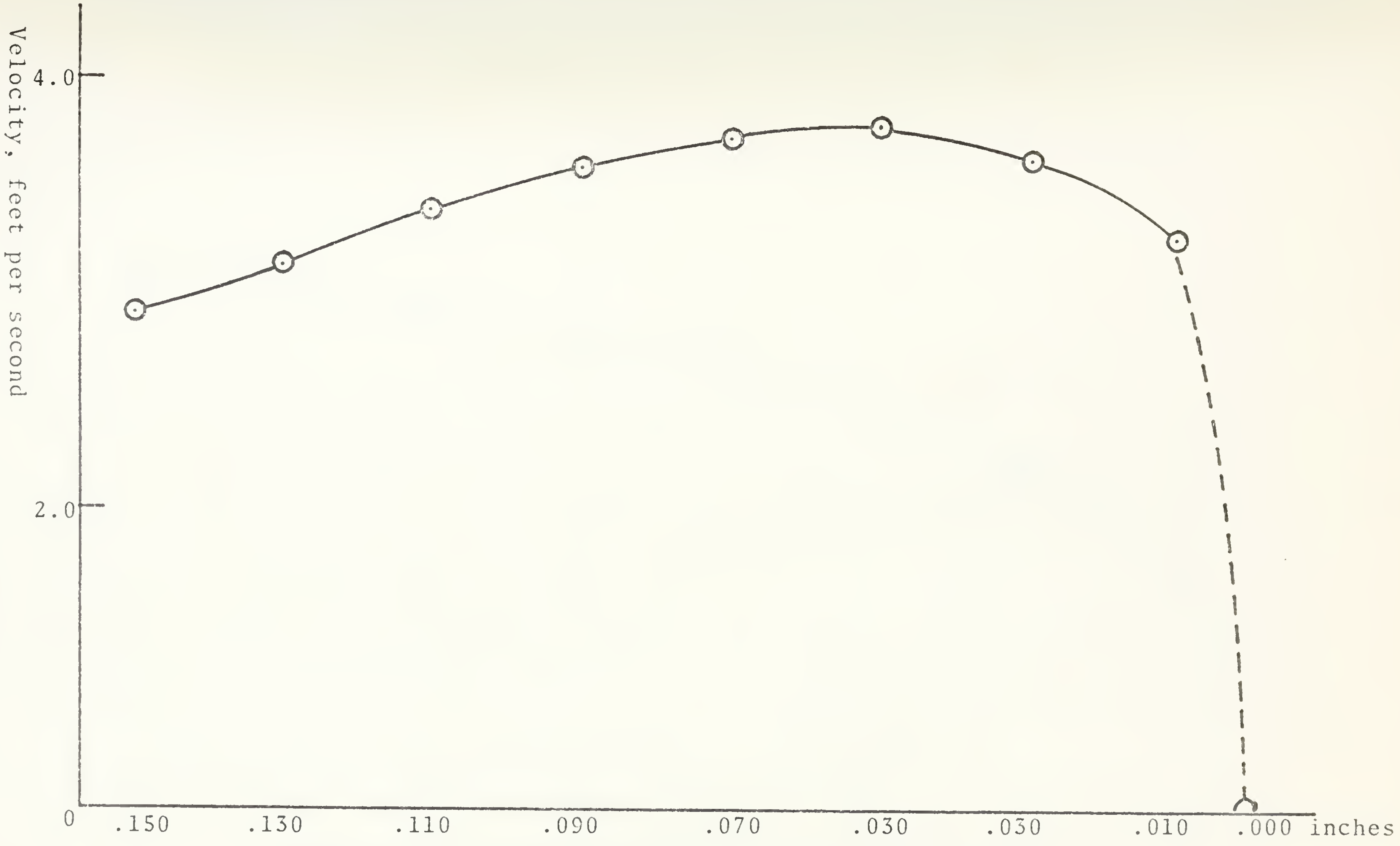

FIGURE 32. Velocity Profile R_e = 1047, Vortex Edge

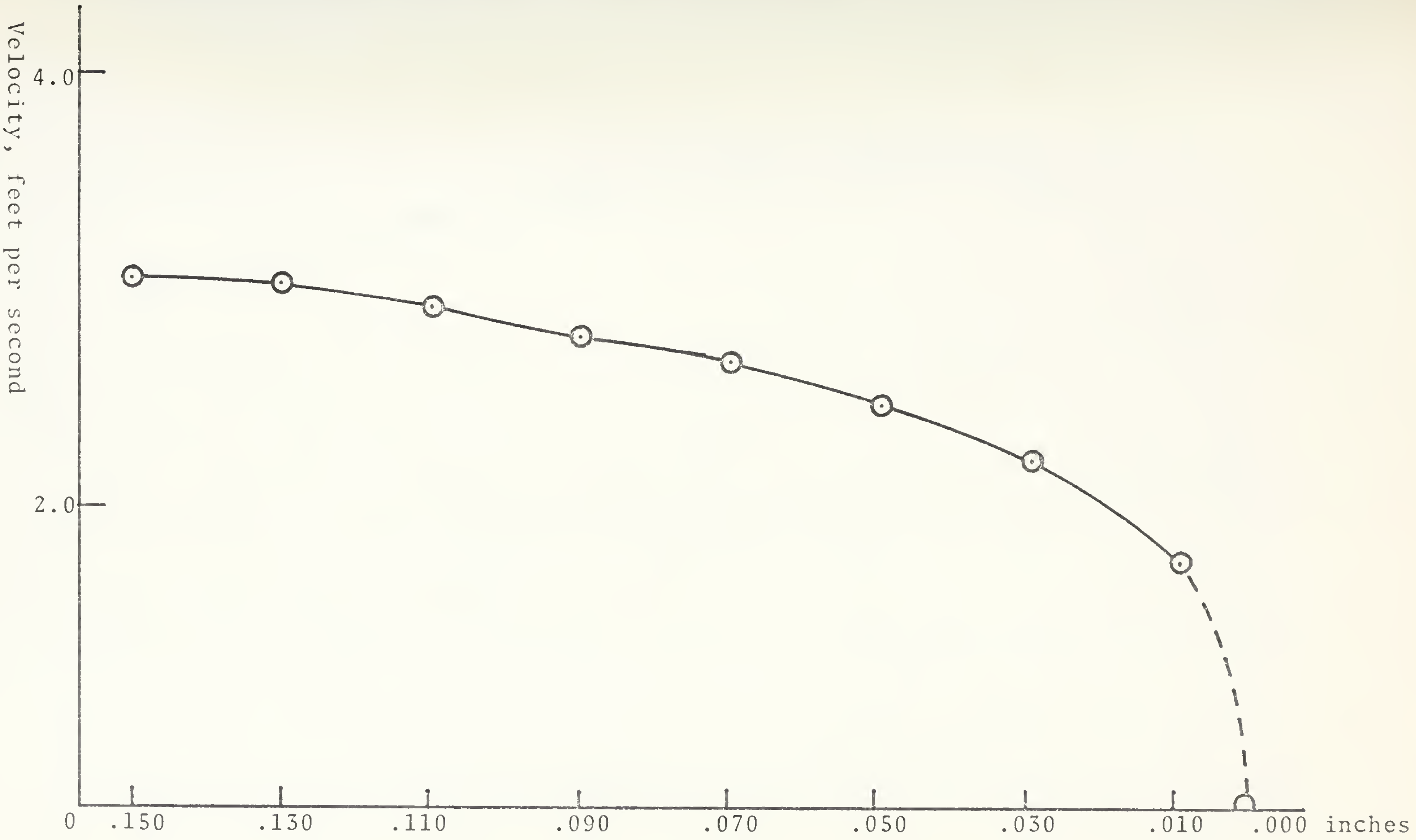

FIGURE 33. Velocity Profile R_e = 1047, Vortex Center

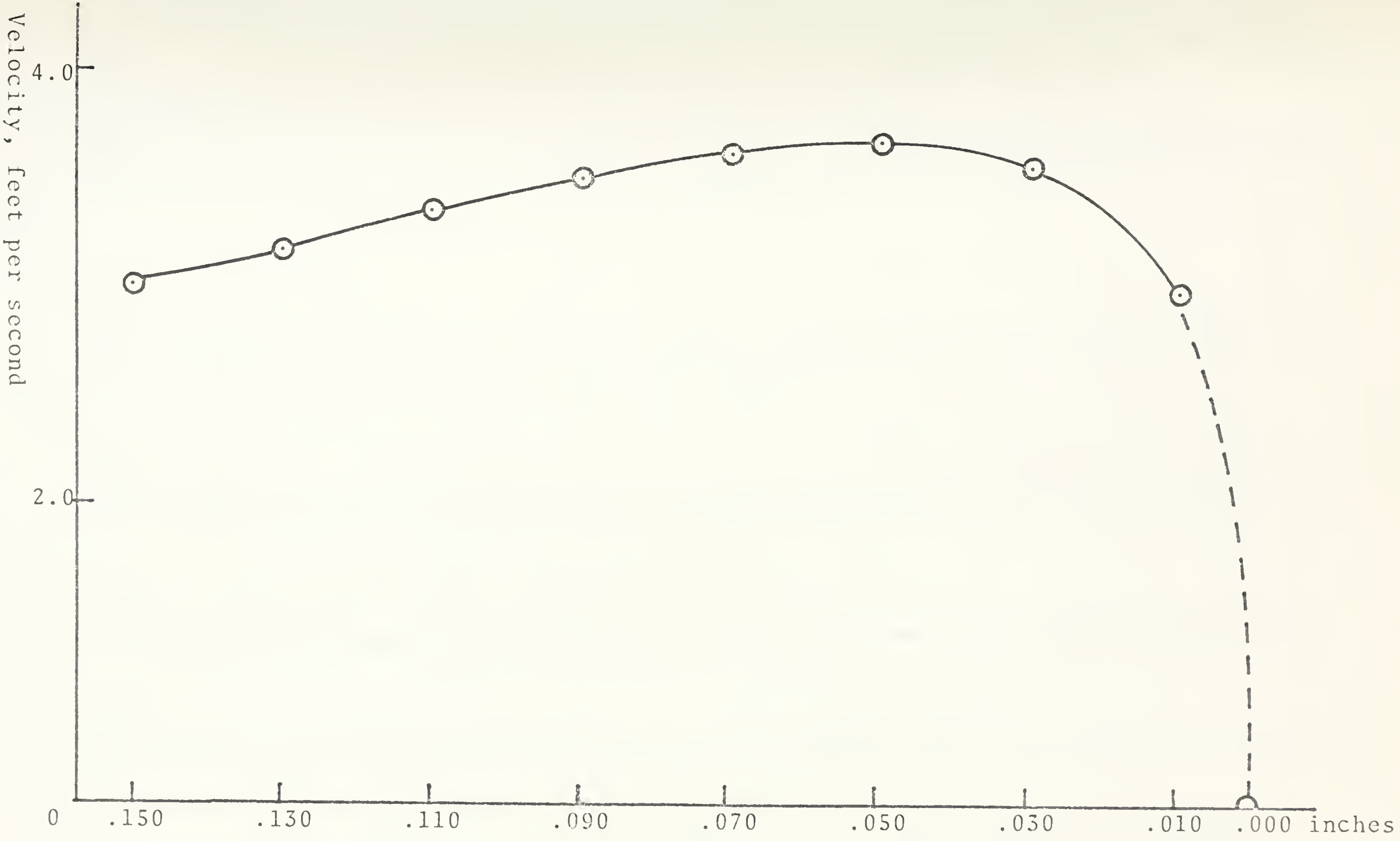

FIGURE 34. Velocity Profile R_e = 1047, Vortex Edge

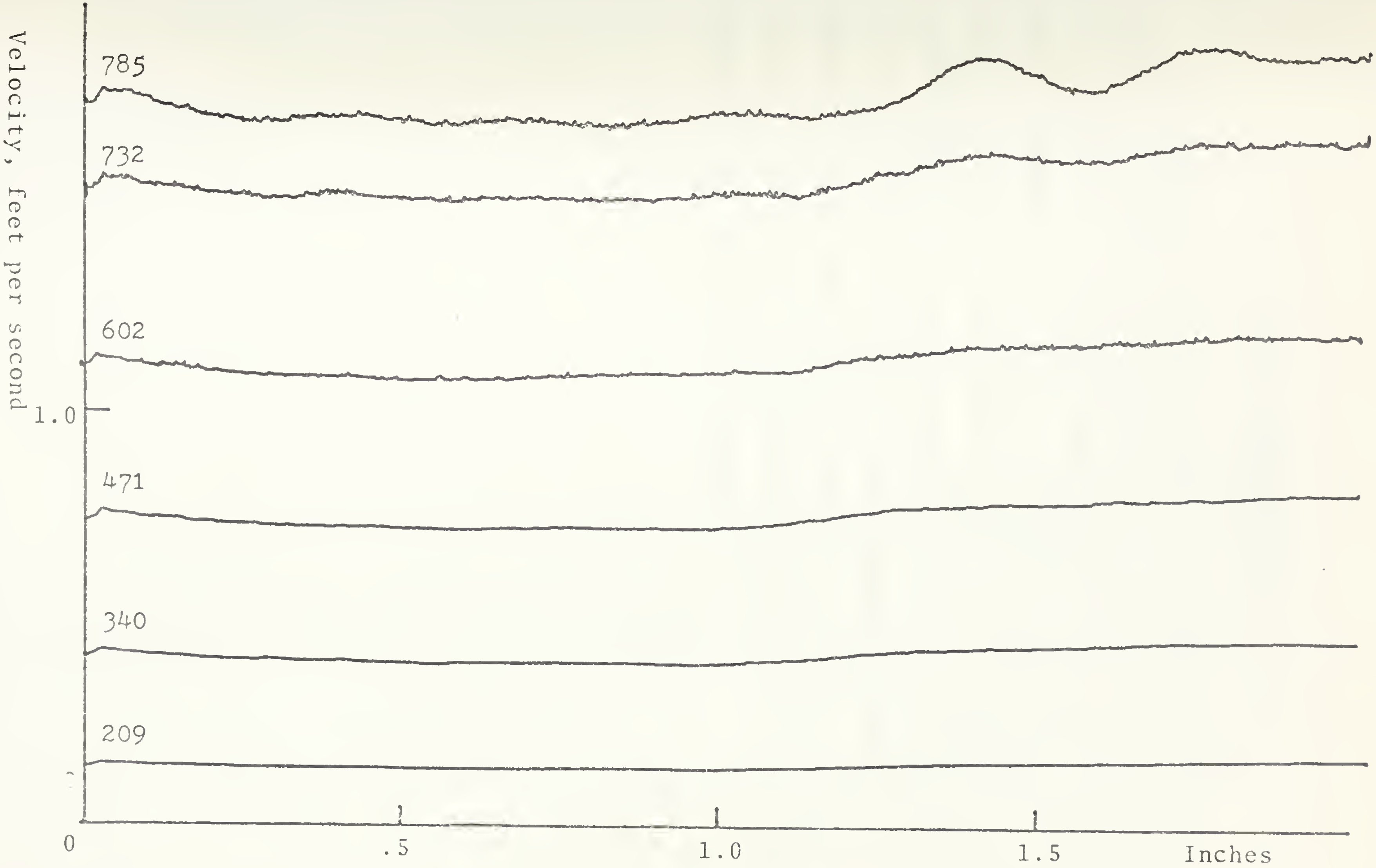

FIGURE 35. Velocity Profile R_e = 209 - 785
Distance from Concave Wall = 0.025 inches.

IV. CONCLUSIONS

The study of velocity profiles resulting from Taylor-Goertler vortices in a curved channel of rectangular cross section has led to the following conclusions. The vortices grew in strength and number as the flow rate increased. It was also concluded that for the particular section of flow investigated the initial formation of the vortices was in agreement with the critical value proposed by Dean. In addition, turbulence in this case did not occur as a step change but rather as a smooth transition growing in a regular manner within the vortex structure.

V. RECOMMENDATIONS

Several recommendations could be made following the completion of this study of Taylor-Goertler vortices. Additional flow visualization could be done to improve the understanding of vortex formation and expansion. Motion pictures would be an improvement in the flow visualization techniques used by McKee.

The next step in anemometer study would be the modification of the test apparatus to permit the investigator to move the probe in a circumferential direction around the curved section in addition to the radial and trans erse movement now available. This additional dimension would permit investigation of vortex growth in relation to distance. Also, rotation of the probe orientation by 90 degrees would allow investigation of the transverse velocity field to be attempted. Hopefully, the vortex field would become sufficiently prominent to overcome concomitant mean flow velocity effects and thus permit closer evaluation of stability criterion.

It was apparent that an improved method of measuring the mean velocity should be developed, permitting a greater resolution for future studies. Improved resolution would permit more exact investigation of critical areas such as initial vortex development and the initiation of turbulence.

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